CSCE 452/752 Fall 2025

16. Asymptotically Optimal Motion Planning

TEXAS A&M UNIVERSITY

AARRG

Introduction

Asymptotically optimal motion planning algorithms improve the quality of the solution over time.

Overview:

- Standard sampling-based motion planning algorithms tend to produce paths that have lots of **extra motion**.
- Asymptotically optimal (AO) motion planning algorithms overcome this problem by **rewiring** the graph as more vertices are added.
- The most widely used AO algorithm is called **RRT***.

Aside: Path smoothing

Path smoothing can help, but its limited by the **homotopy class** of the initial solution.

RRT*: Main idea

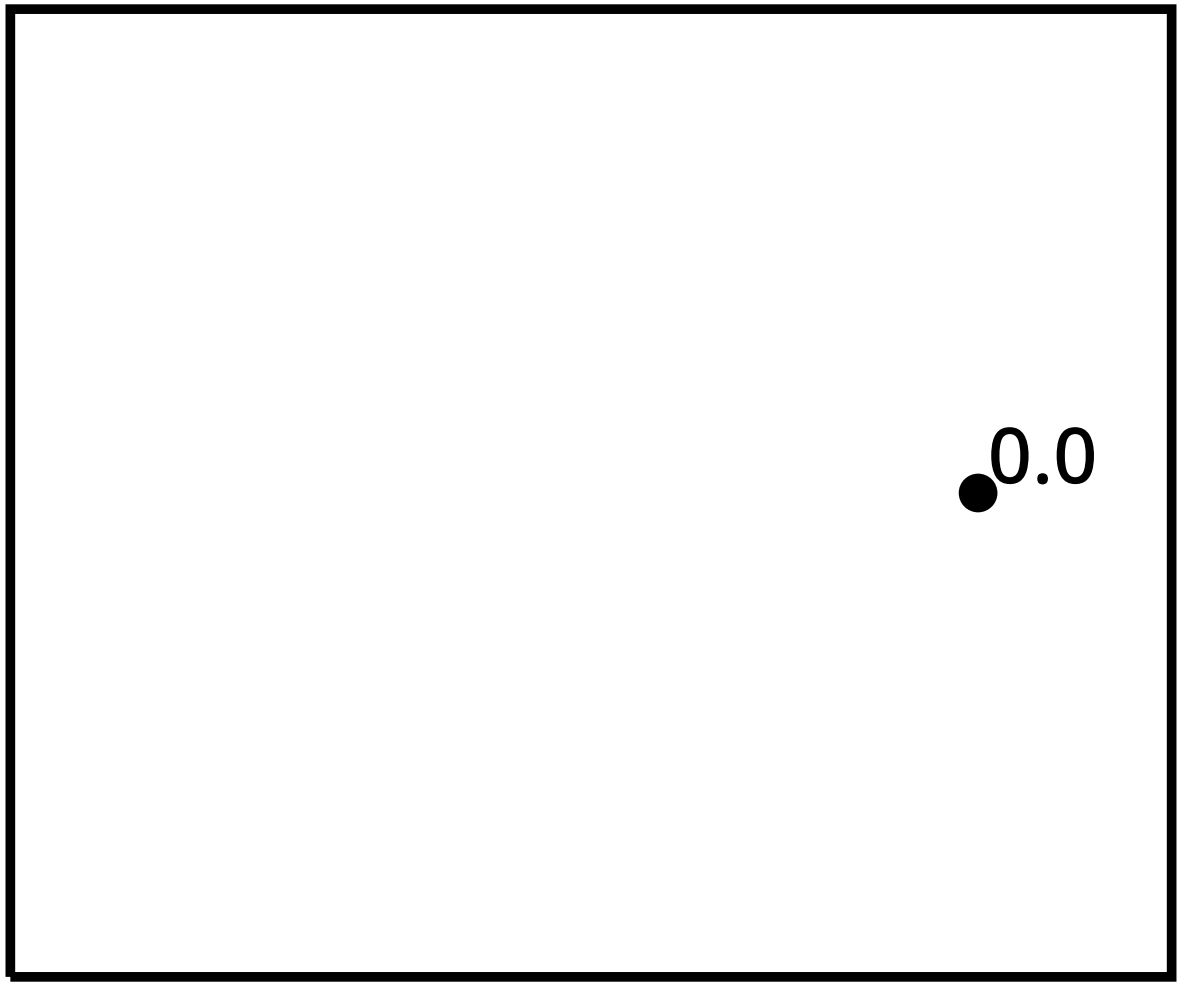
RRT* differs from RRT in two ways:

Change 1: Each new node connects to the **best** nearby parent, determined by the new node's distance to the root. Not necessarily the node that was nearest neighbor.

Change 2: For each new node, check nearby nodes to see if any would be have smaller distances with the new node as their parent. If so, **rewire** the tree.

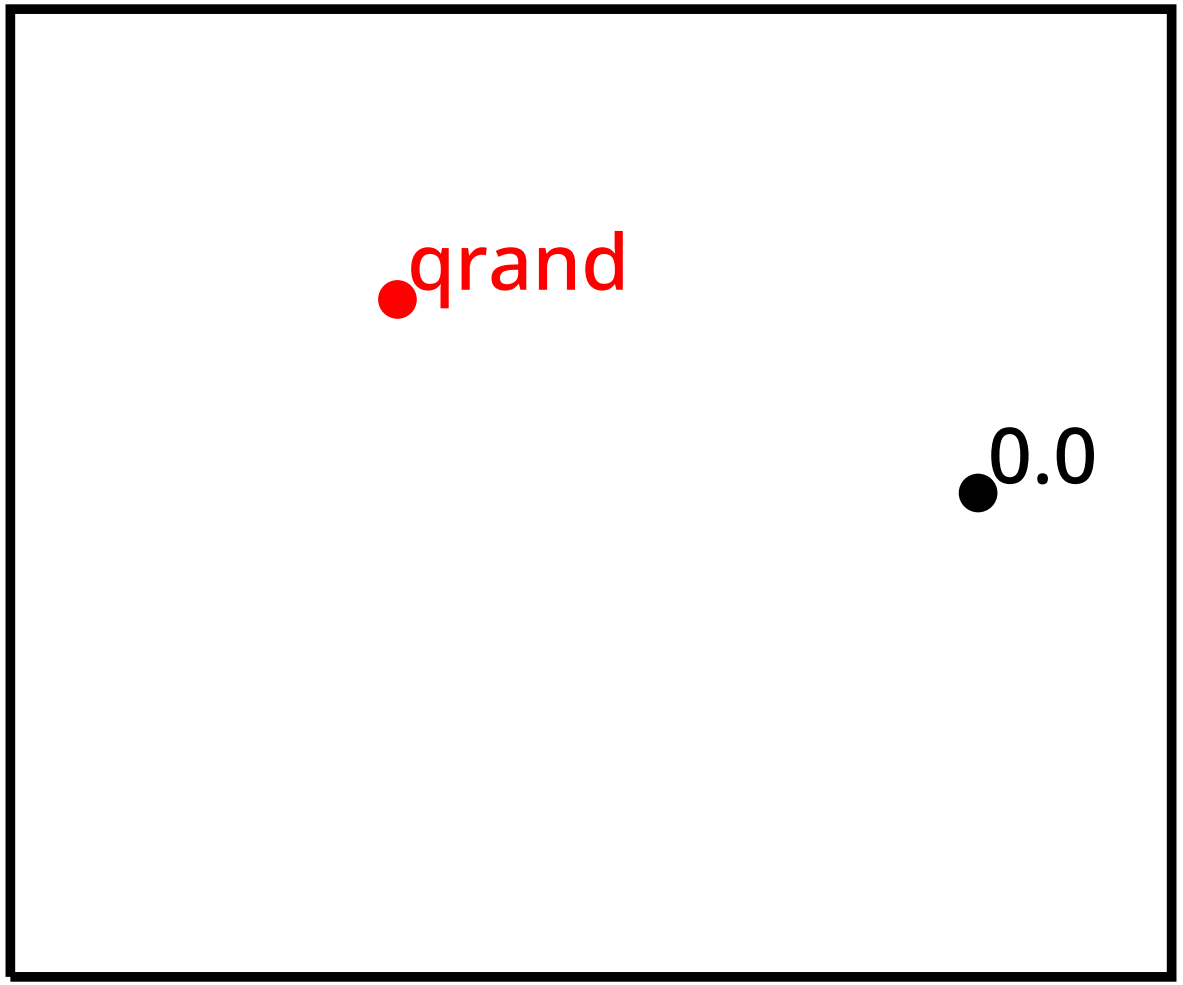
RRT*: Example 1

Start from a tree with just the start configuration.



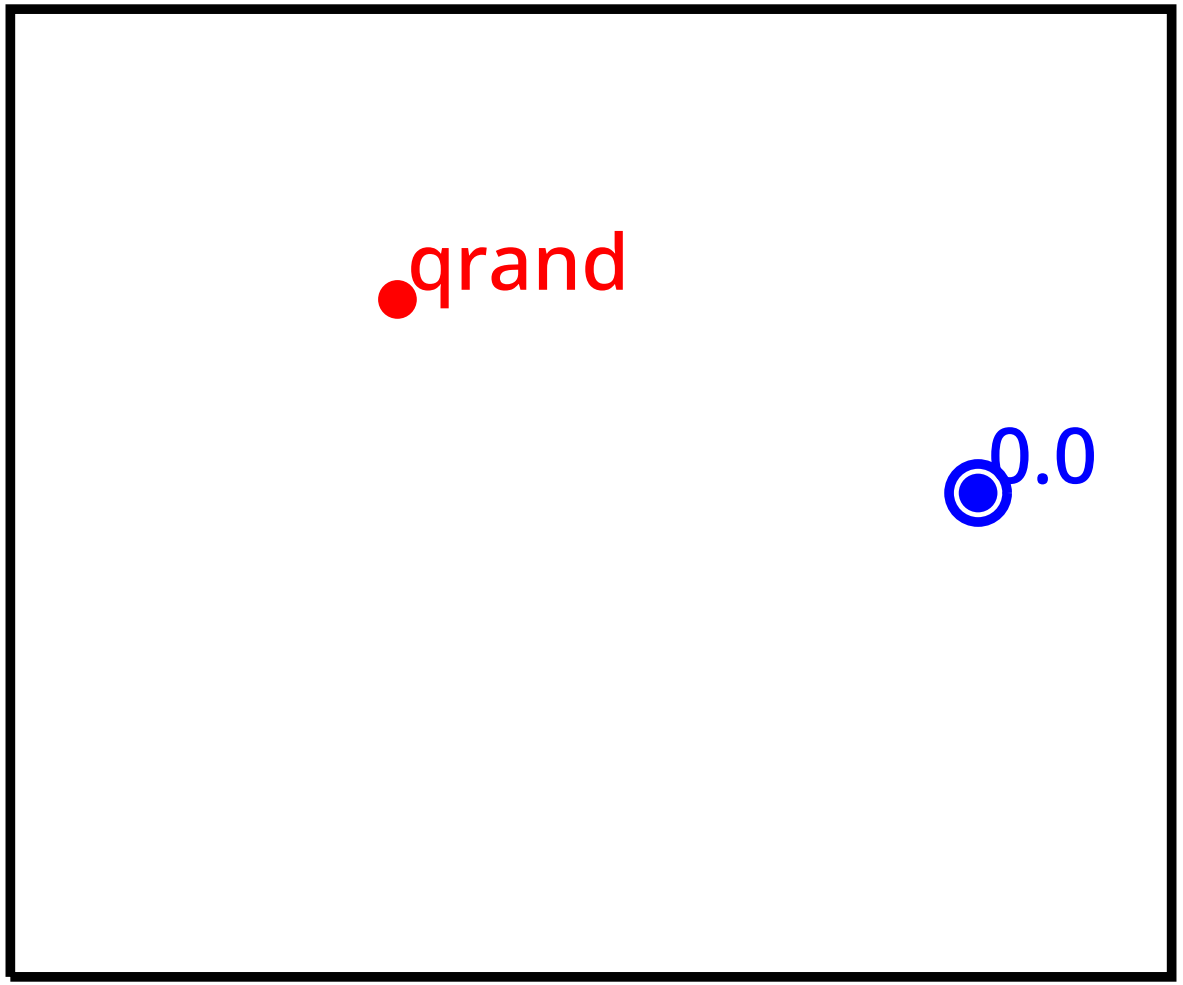
RRT*: Example 1

Choose a random sample.



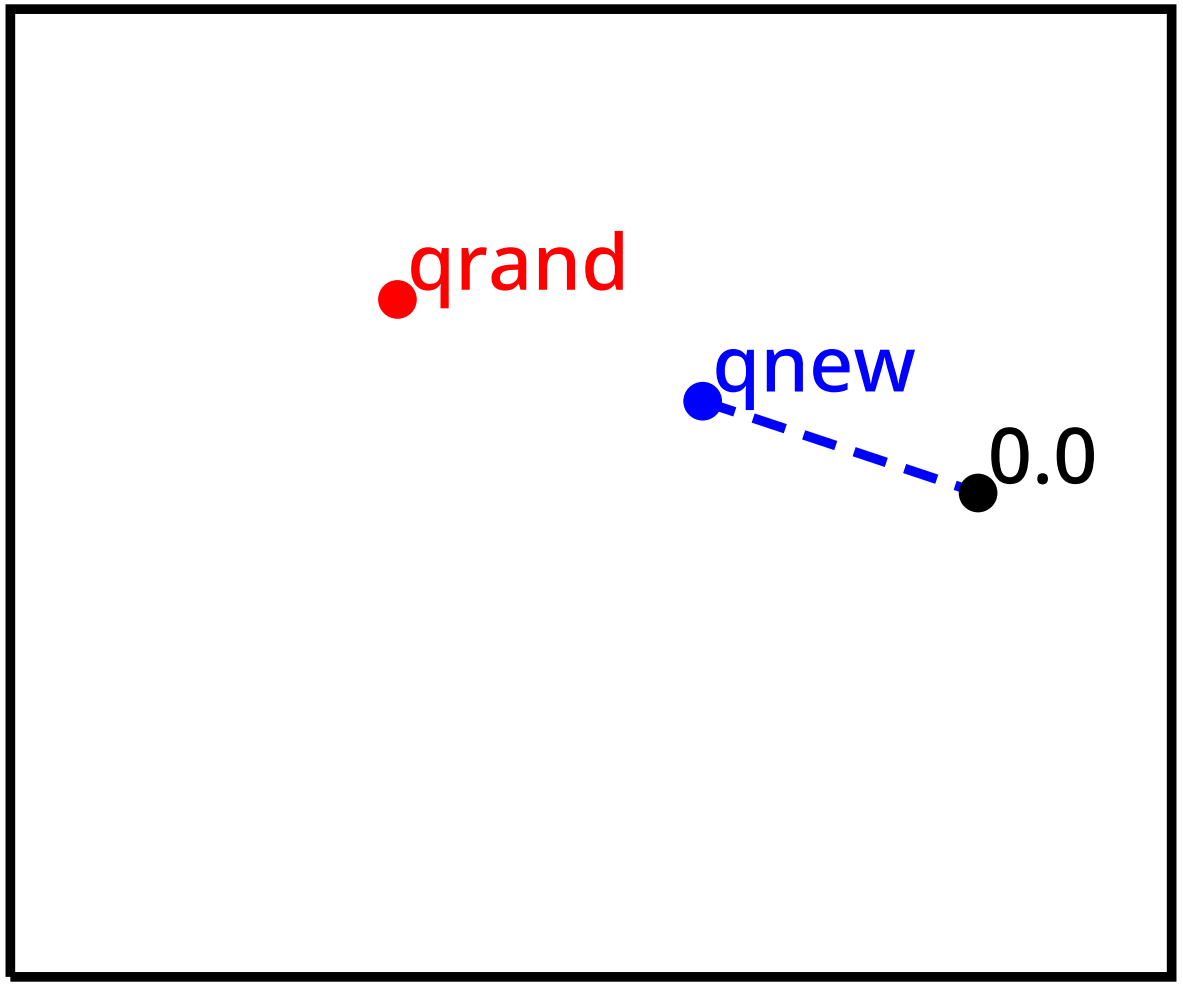
RRT*: Example 1

Find the nearest neighbor.



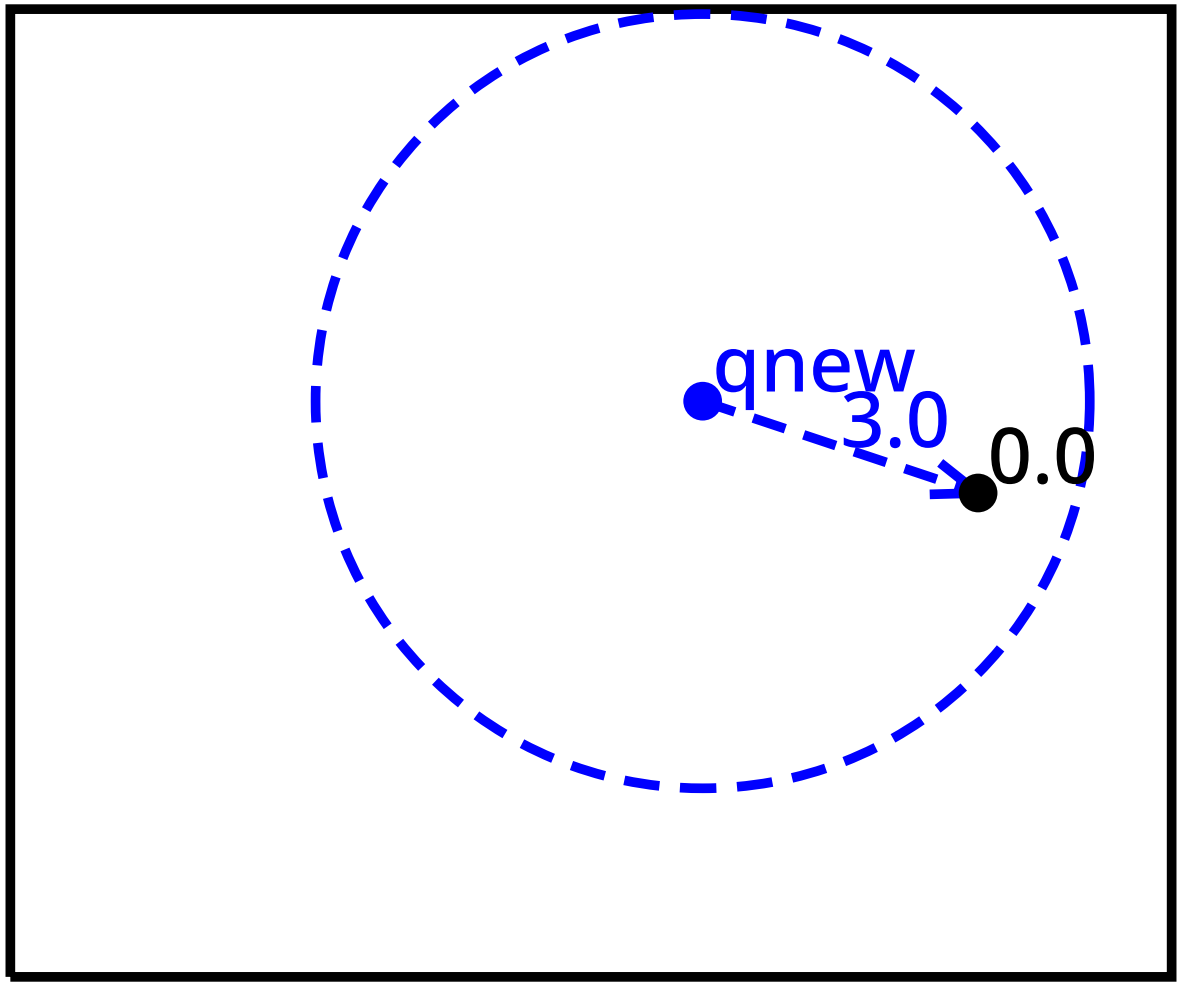
RRT*: Example 1

Extend from the nearest neighbor toward the sample.



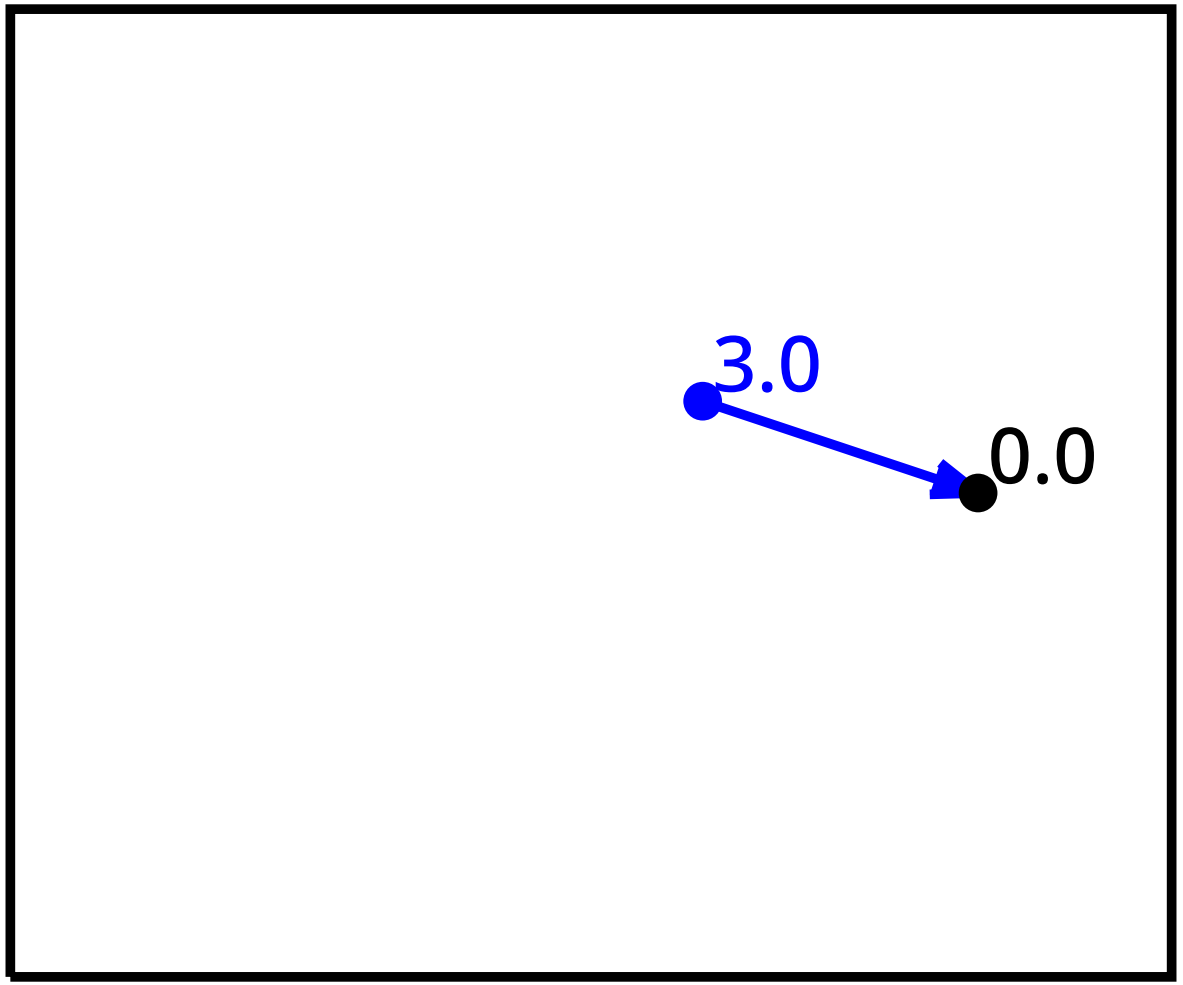
RRT*: Example 1

Find candidate parents for the new node; 1 candidate this time.



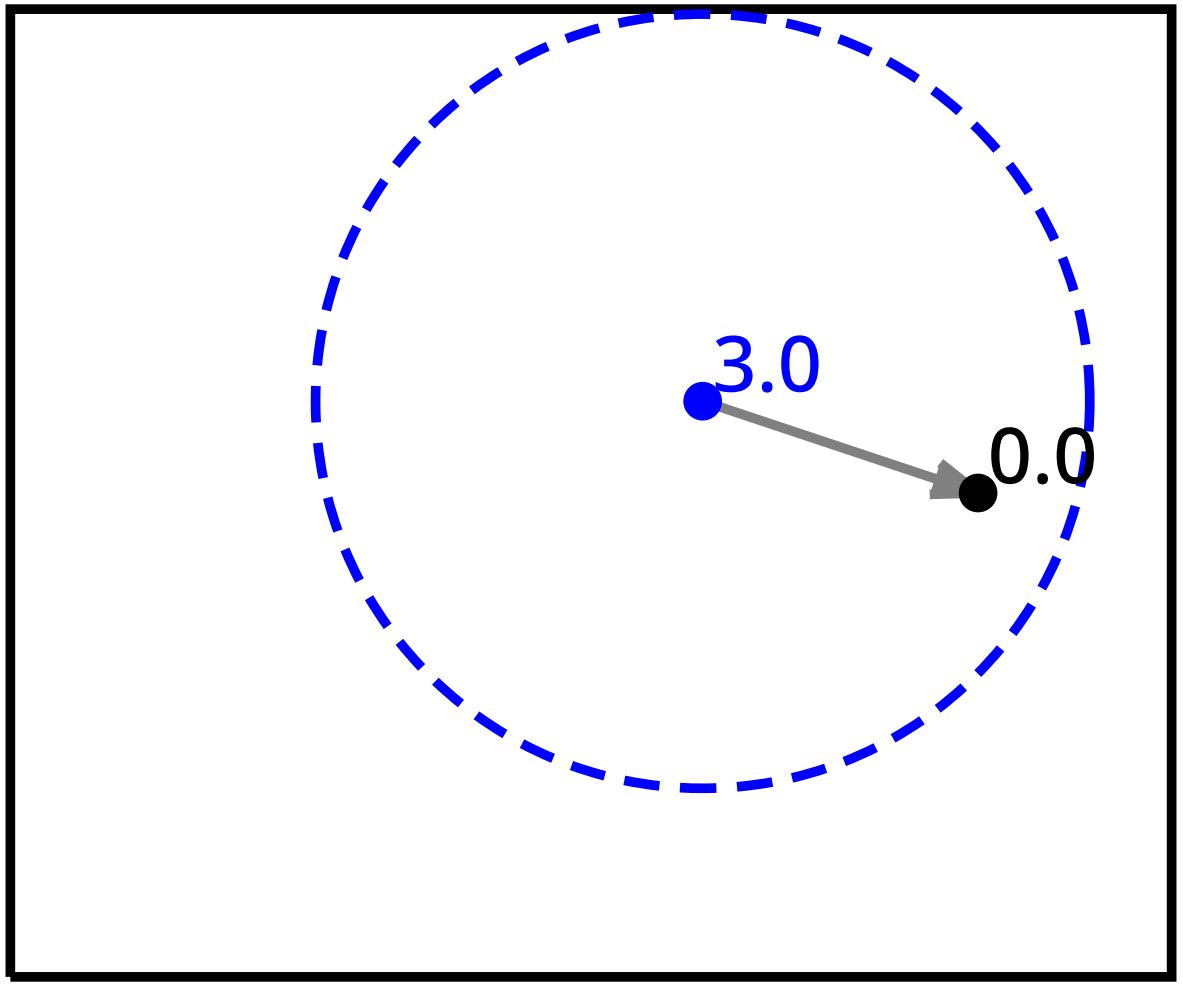
RRT*: Example 1

Choose the best parent for the new node.



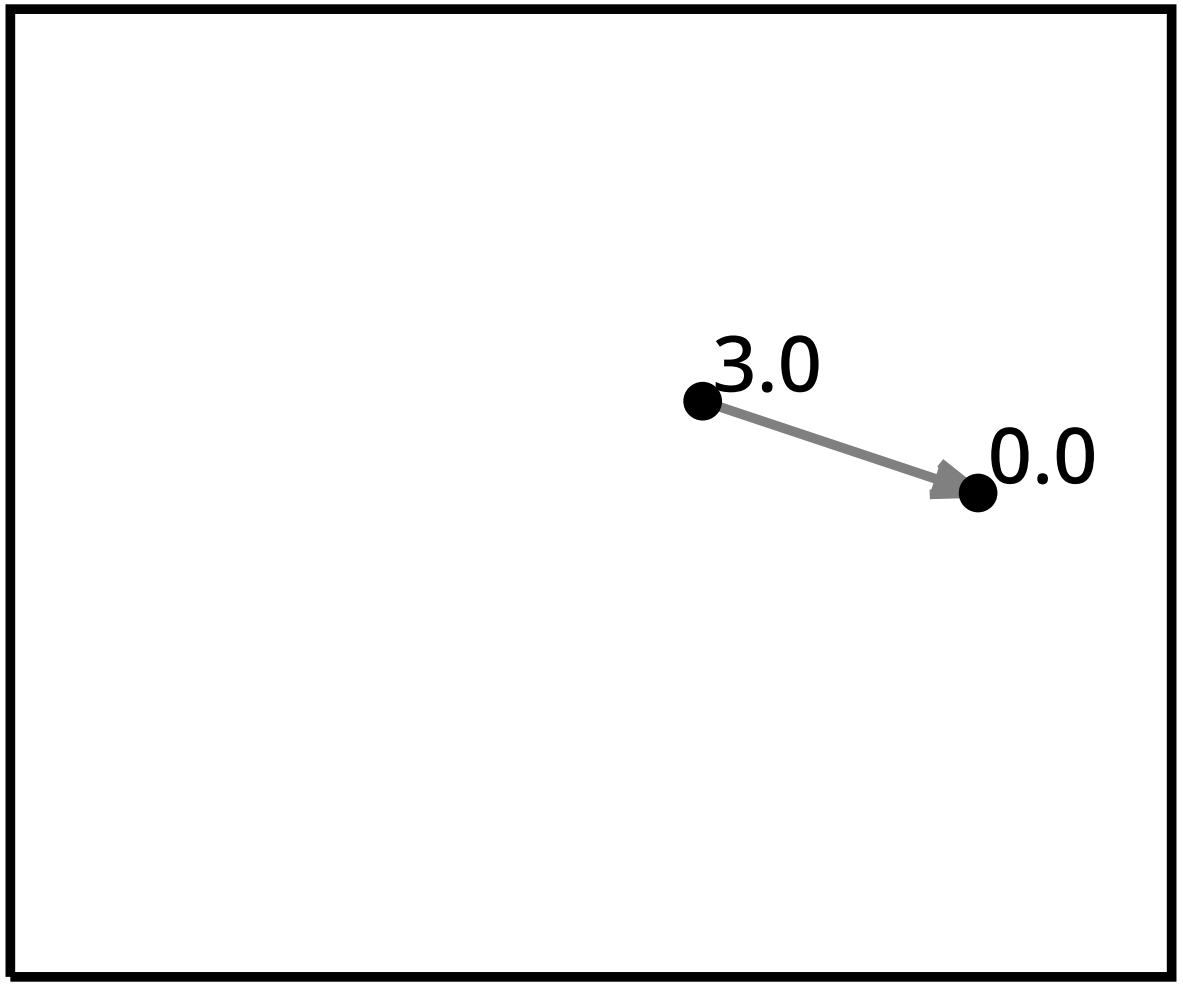
RRT*: Example 1

Find rewiring candidates near the new node; 0 candidates this time.



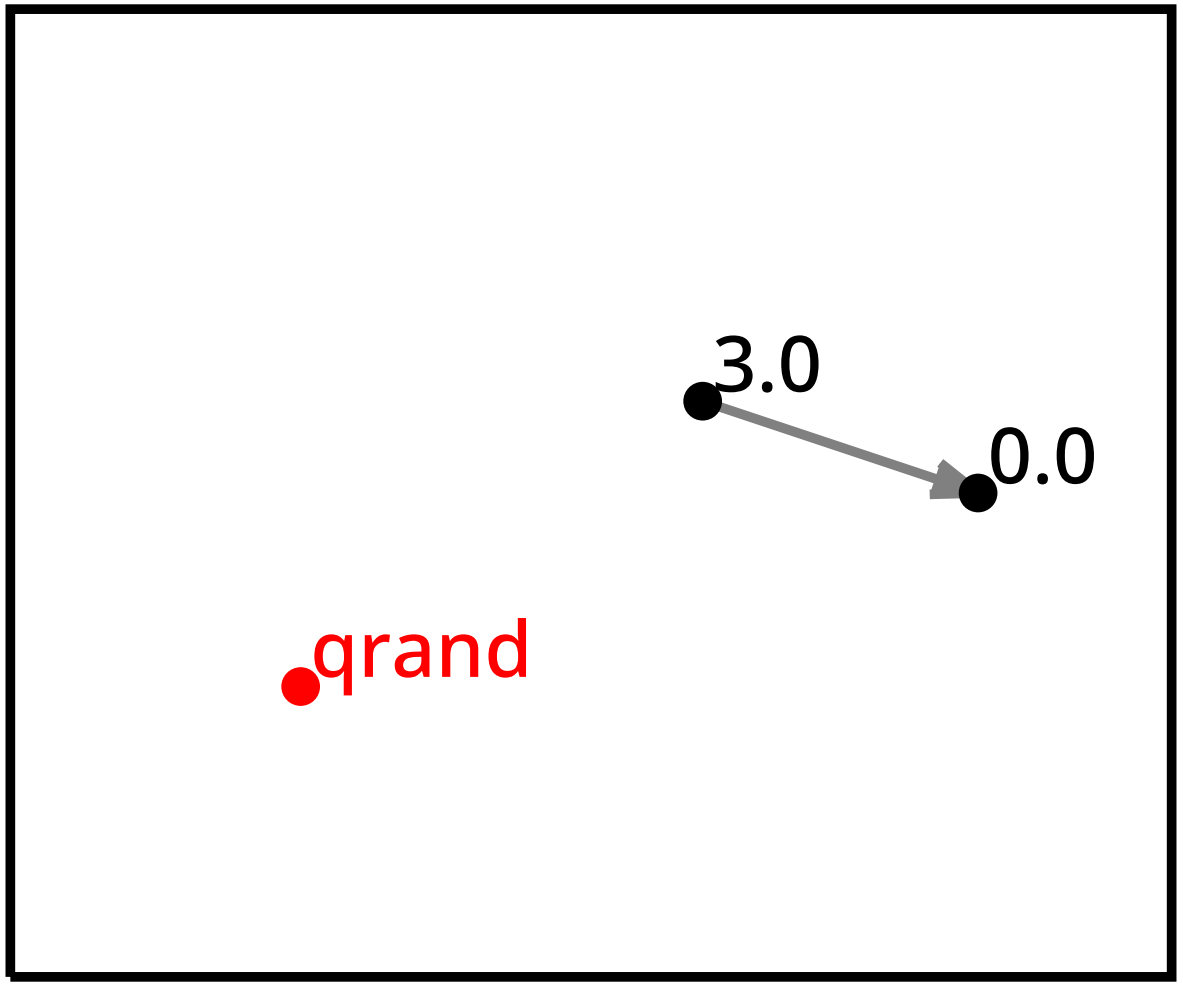
RRT*: Example 1

Rewire if needed; 0 changes this time.



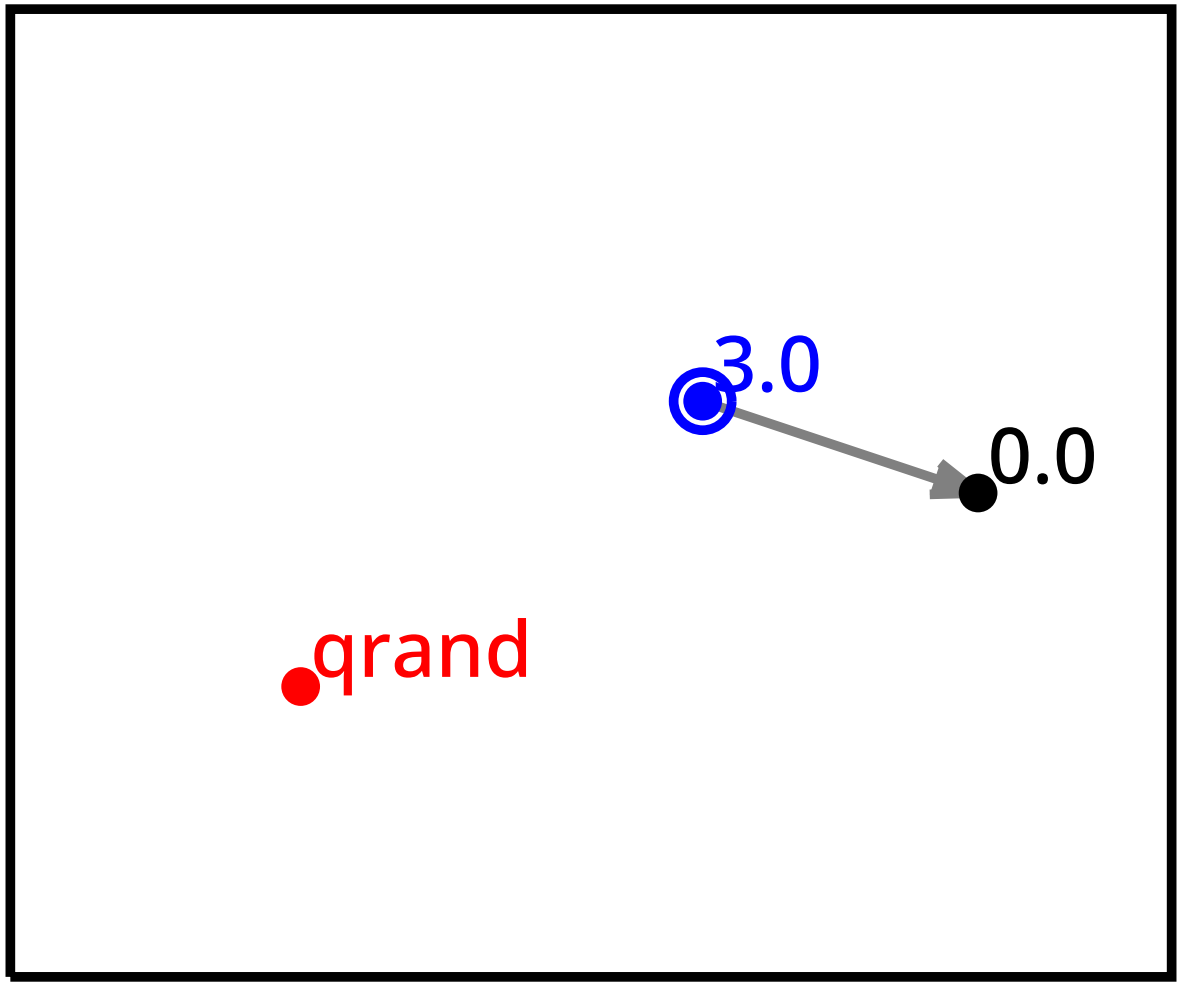
RRT*: Example 1

Choose a random sample.



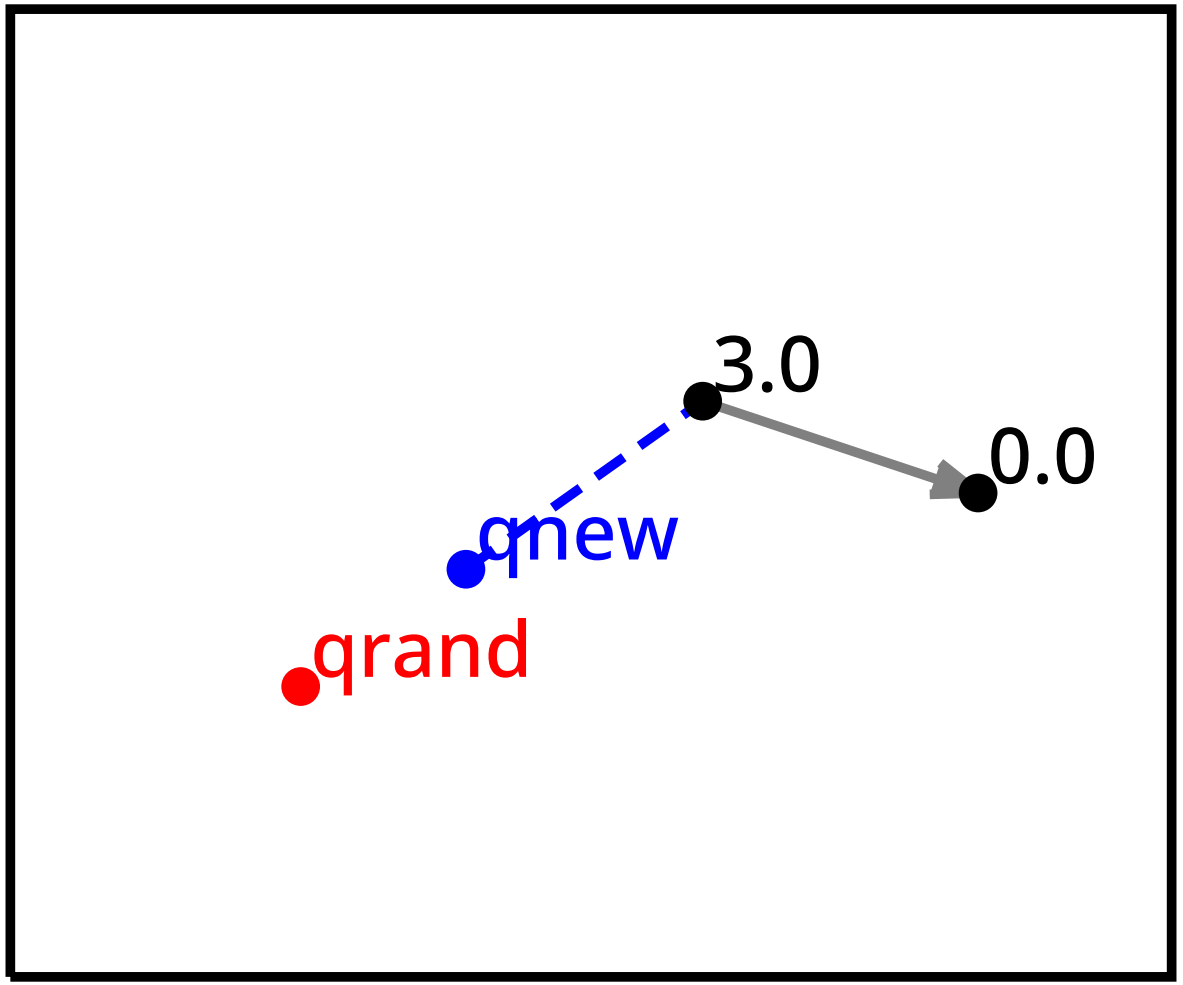
RRT*: Example 1

Find the nearest neighbor.



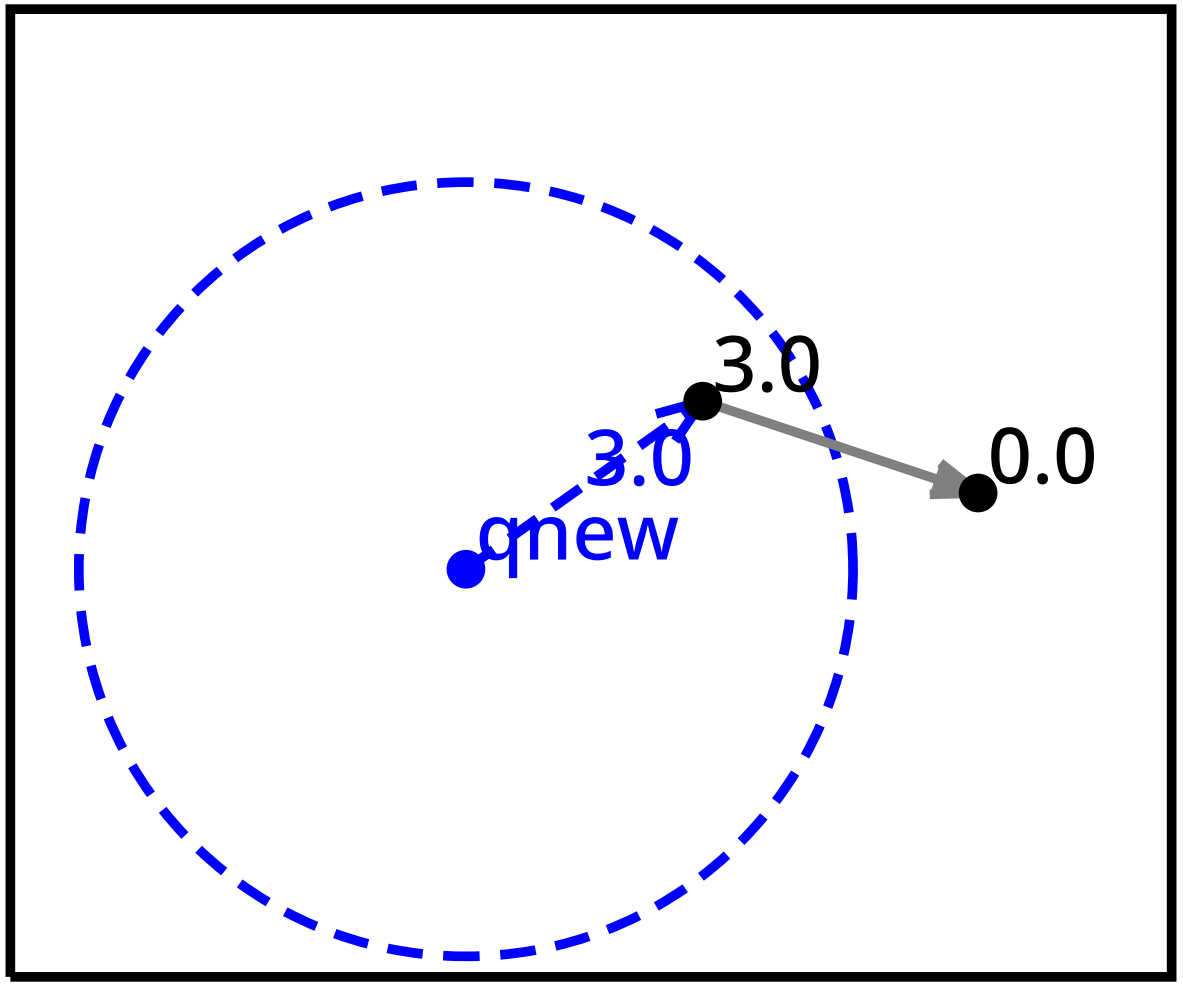
RRT*: Example 1

Extend from the nearest neighbor toward the sample.



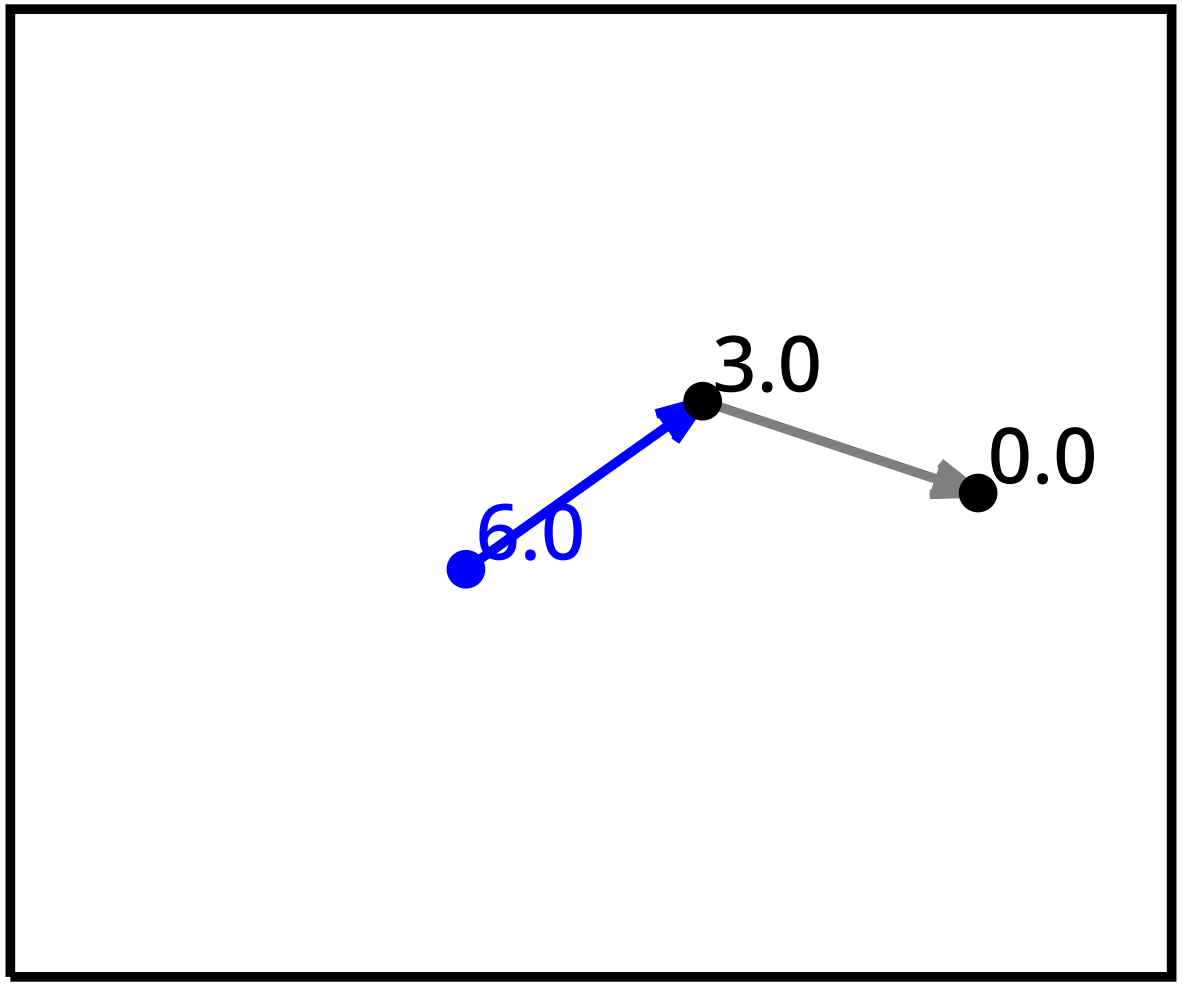
RRT*: Example 1

Find candidate parents for the new node; 1 candidate this time.



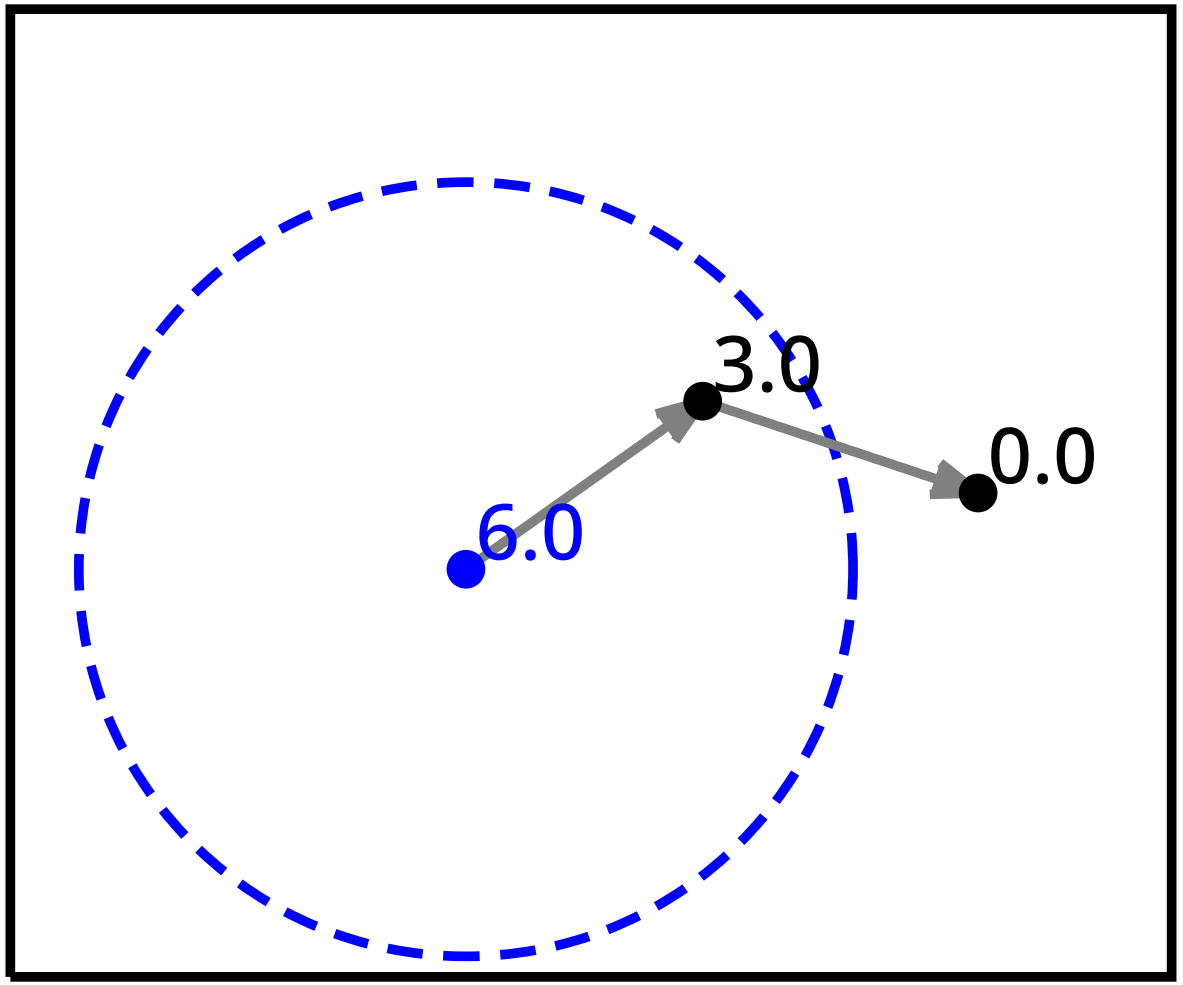
RRT*: Example 1

Choose the best parent for the new node.



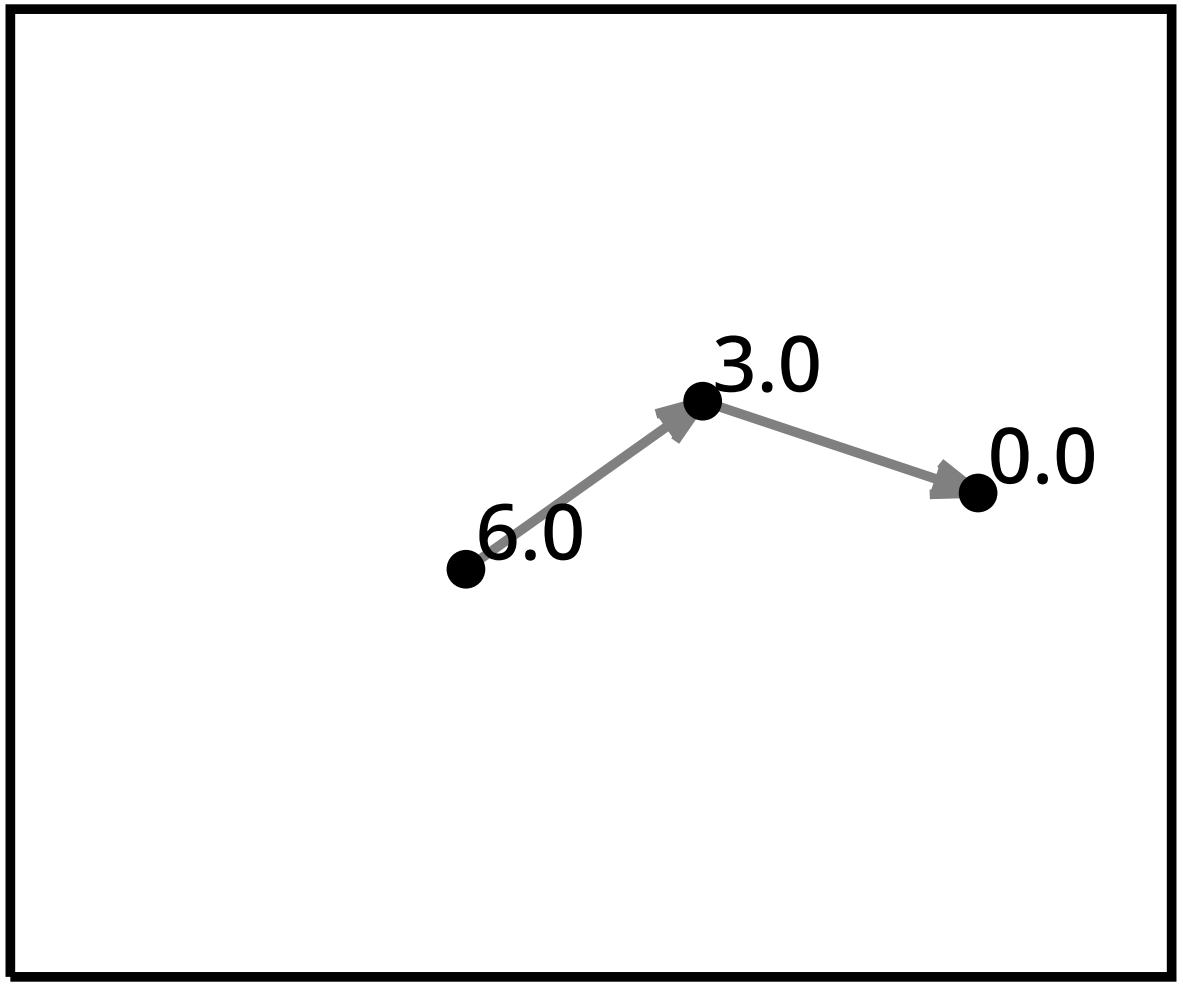
RRT*: Example 1

Find rewiring candidates near the new node; 0 candidates this time.



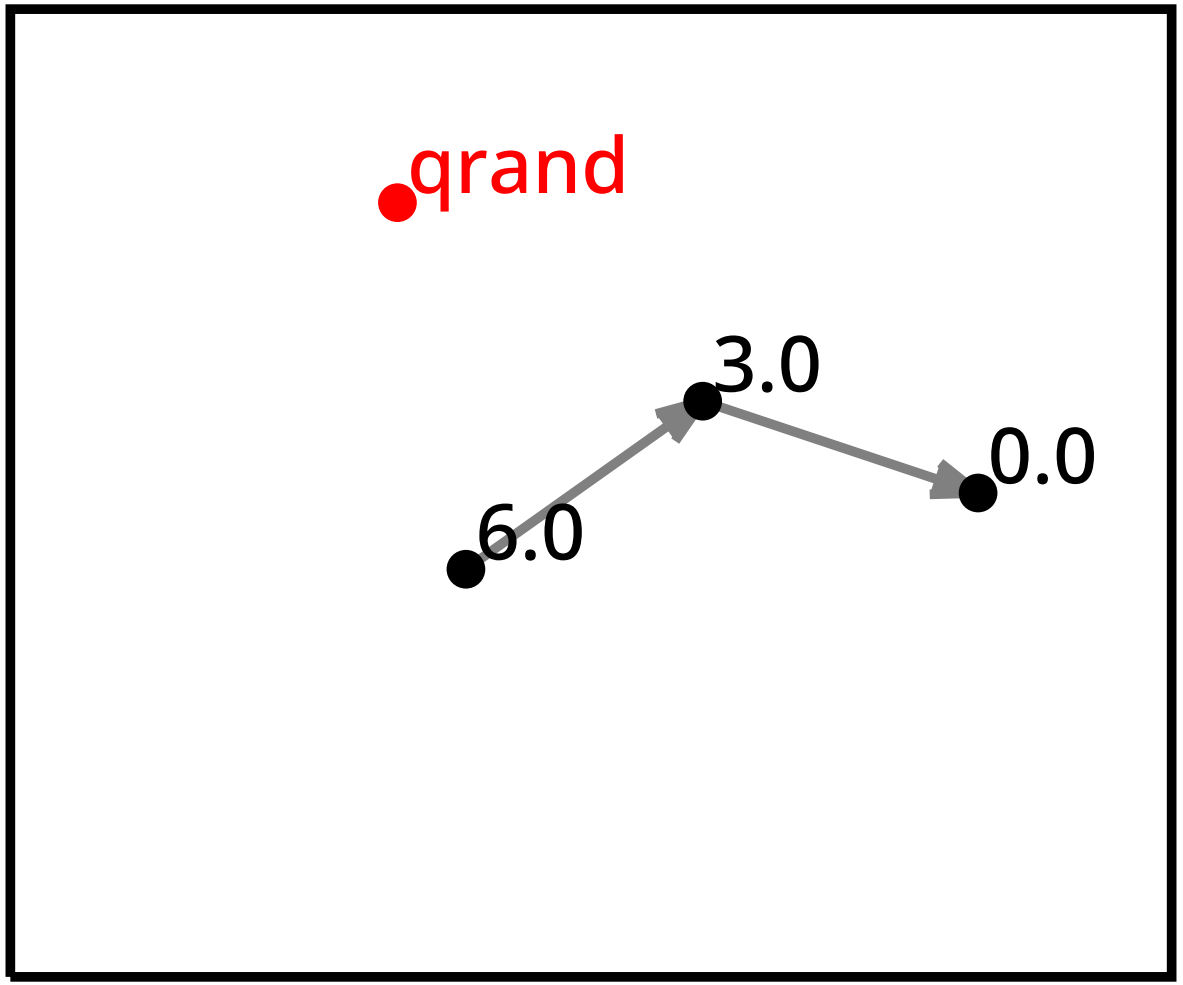
RRT*: Example 1

Rewire if needed; 0 changes this time.



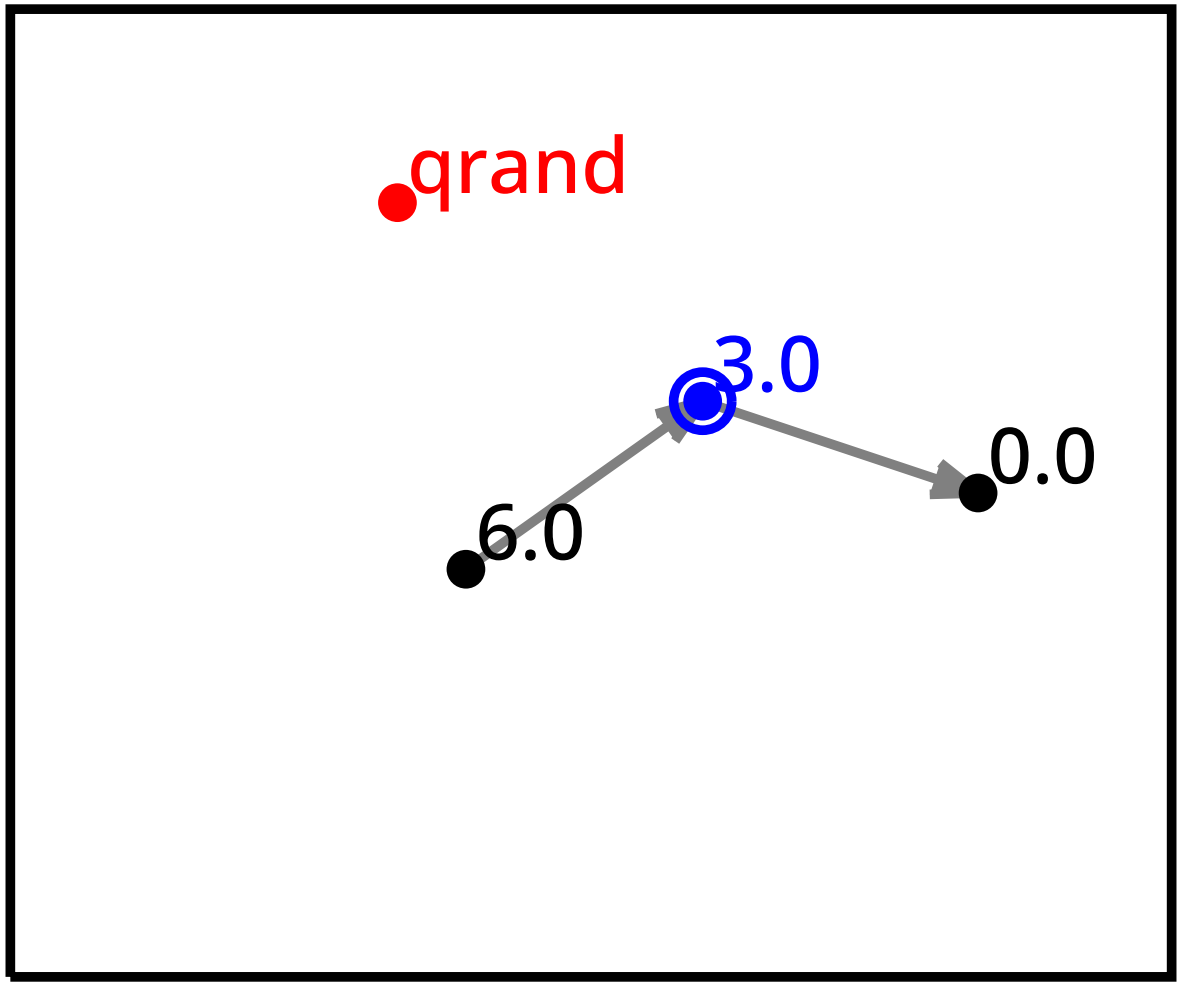
RRT*: Example 1

Choose a random sample.



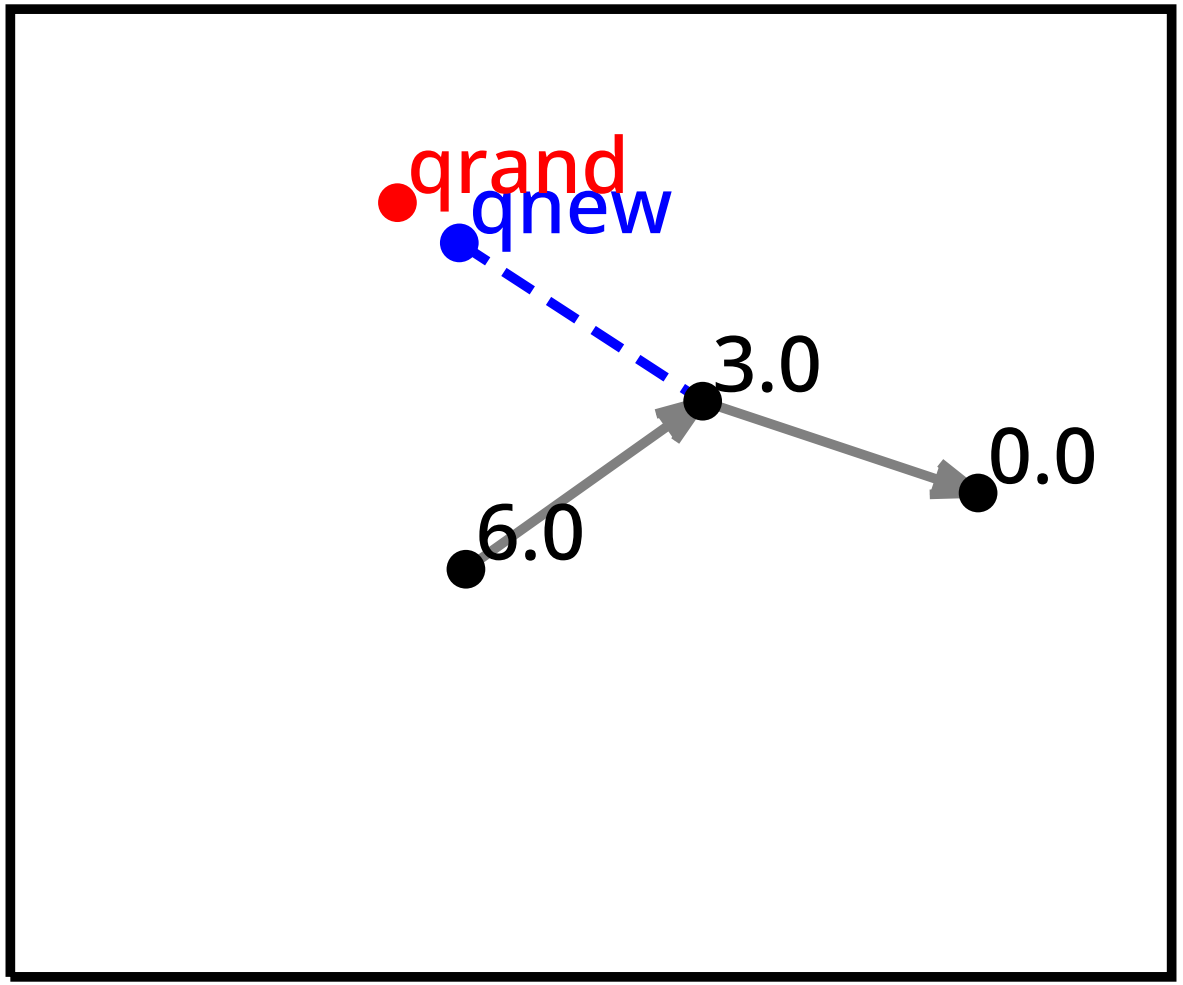
RRT*: Example 1

Find the nearest neighbor.



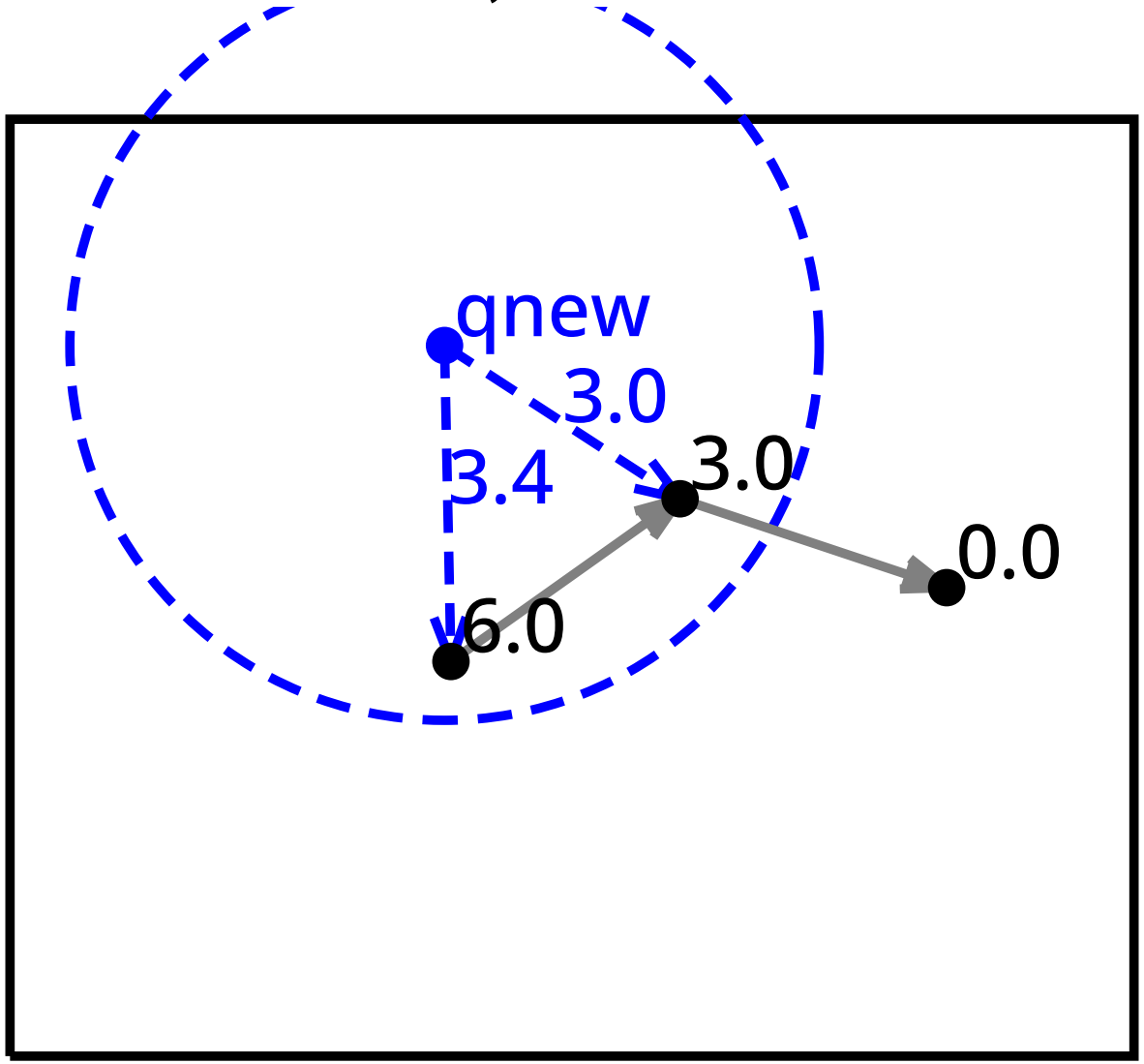
RRT*: Example 1

Extend from the nearest neighbor toward the sample.



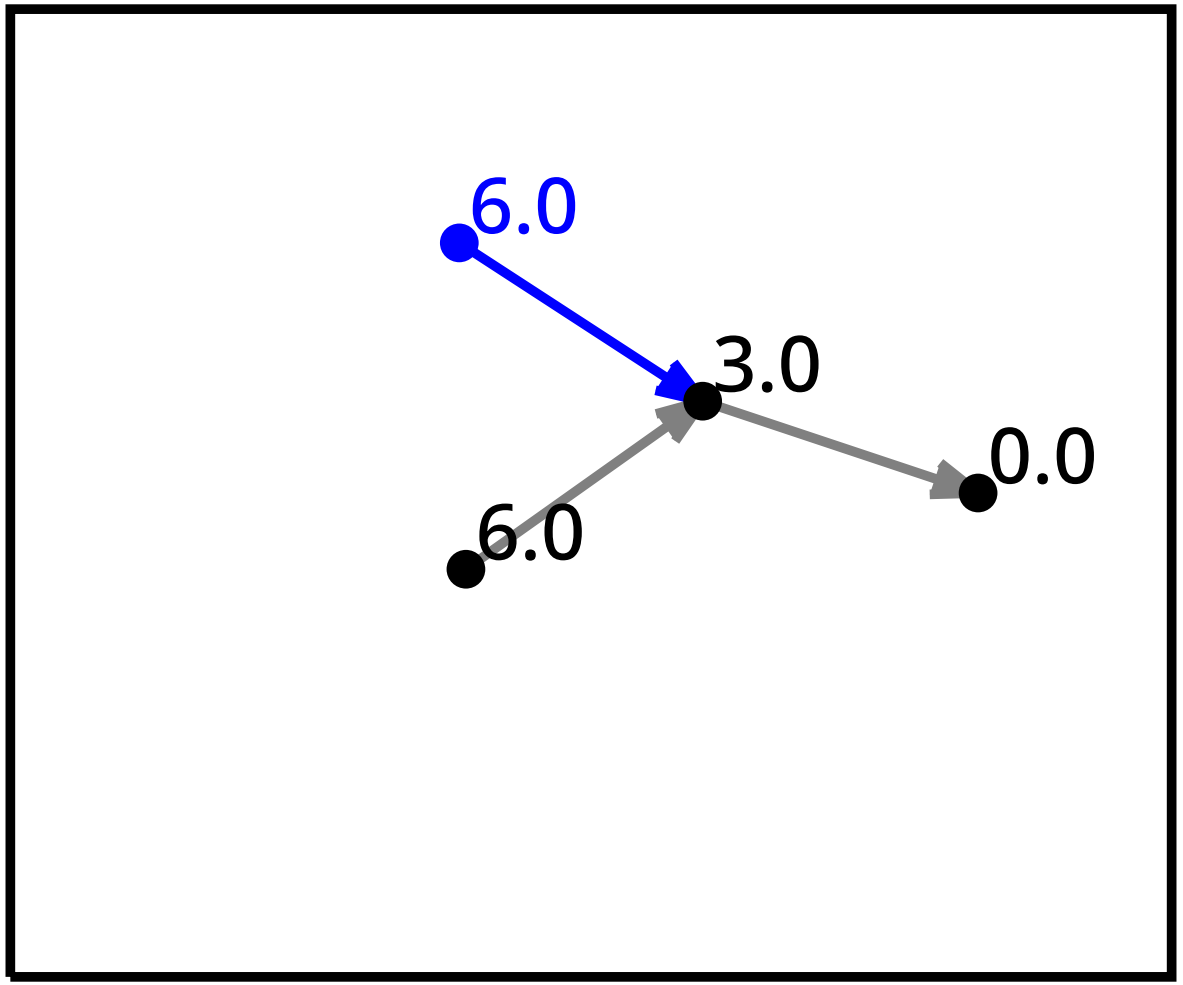
RRT*: Example 1

Find candidate parents for the new node; 2 candidates this time.



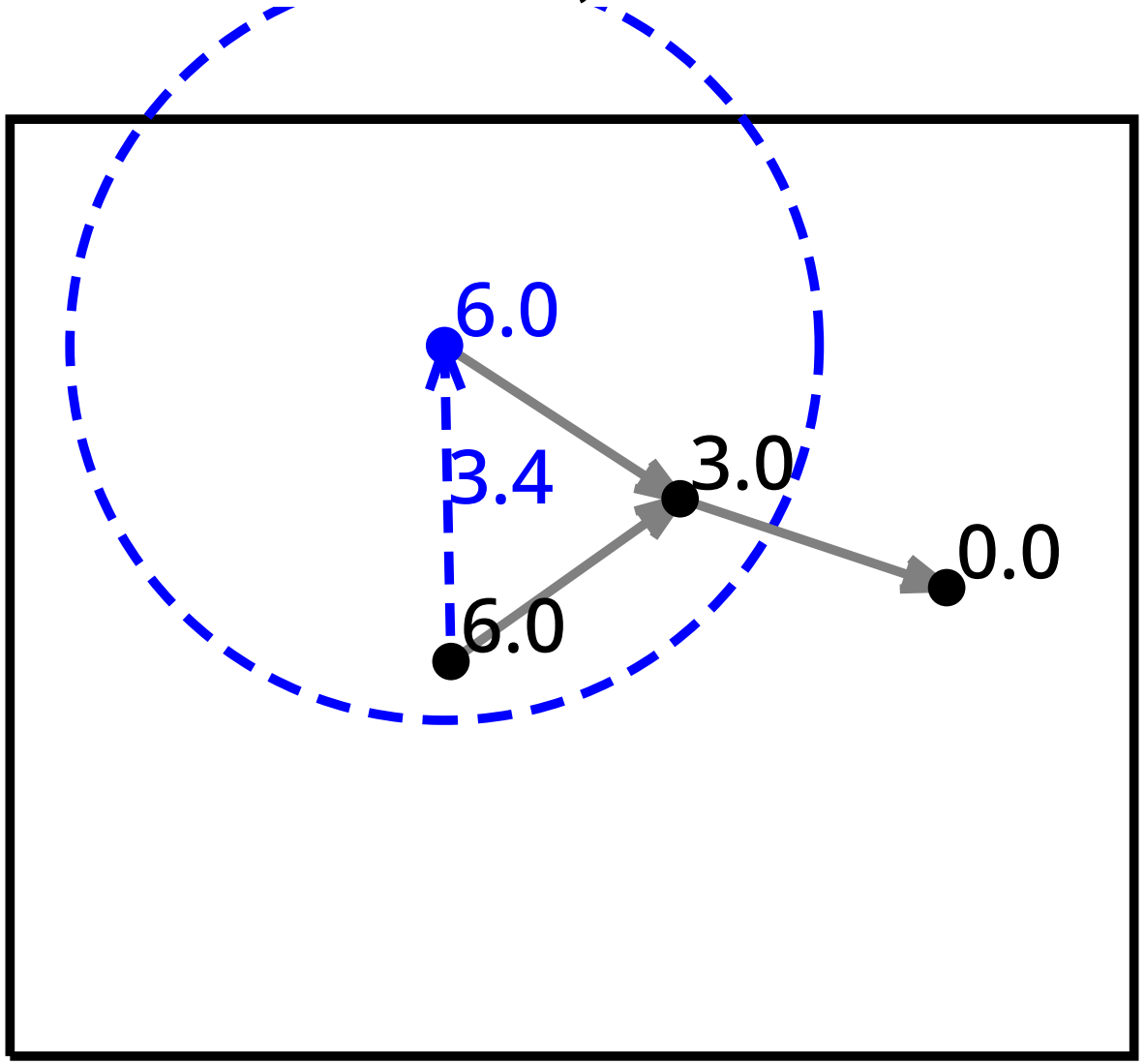
RRT*: Example 1

Choose the best parent for the new node.



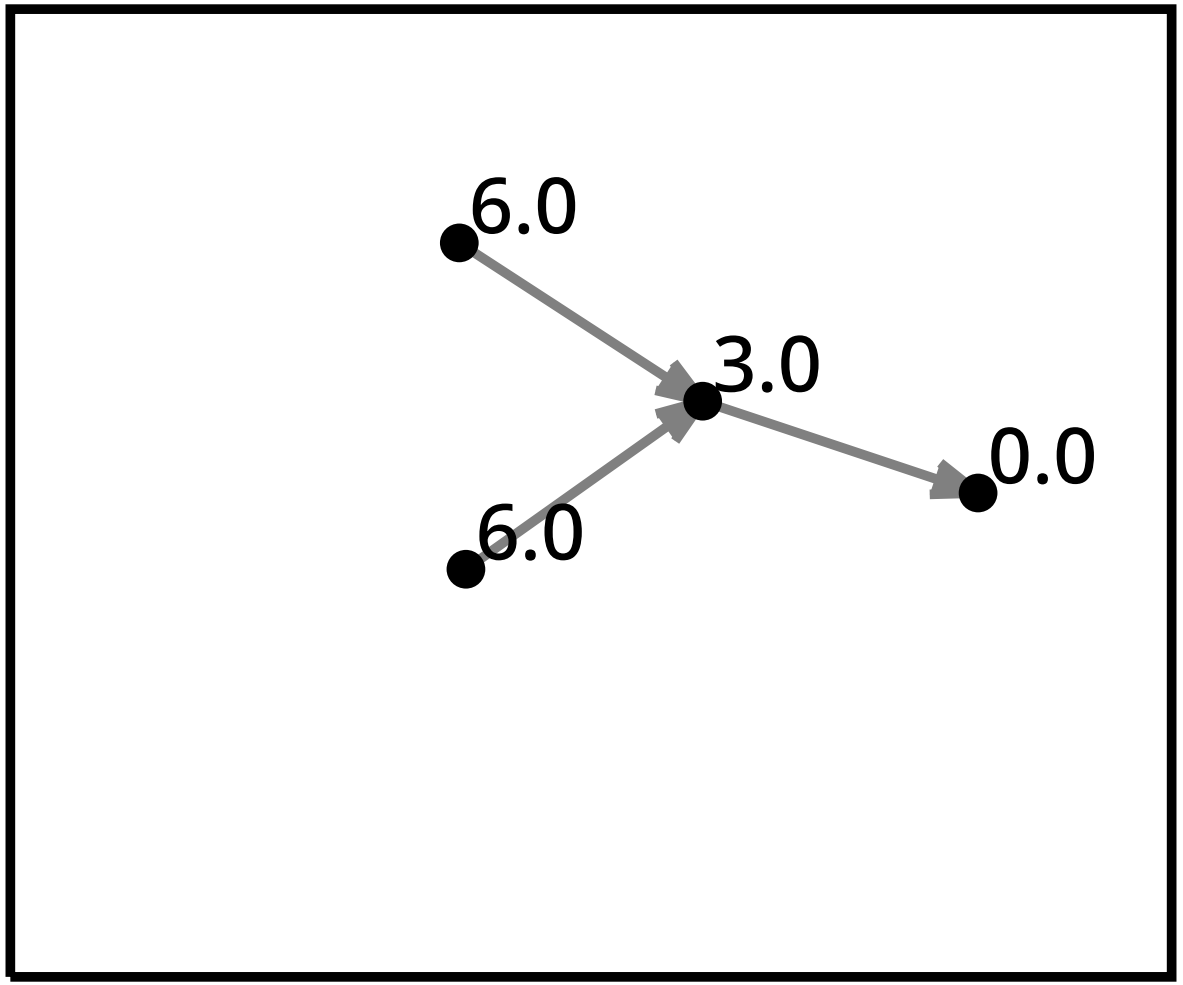
RRT*: Example 1

Find rewiring candidates near the new node; 1 candidate this time.



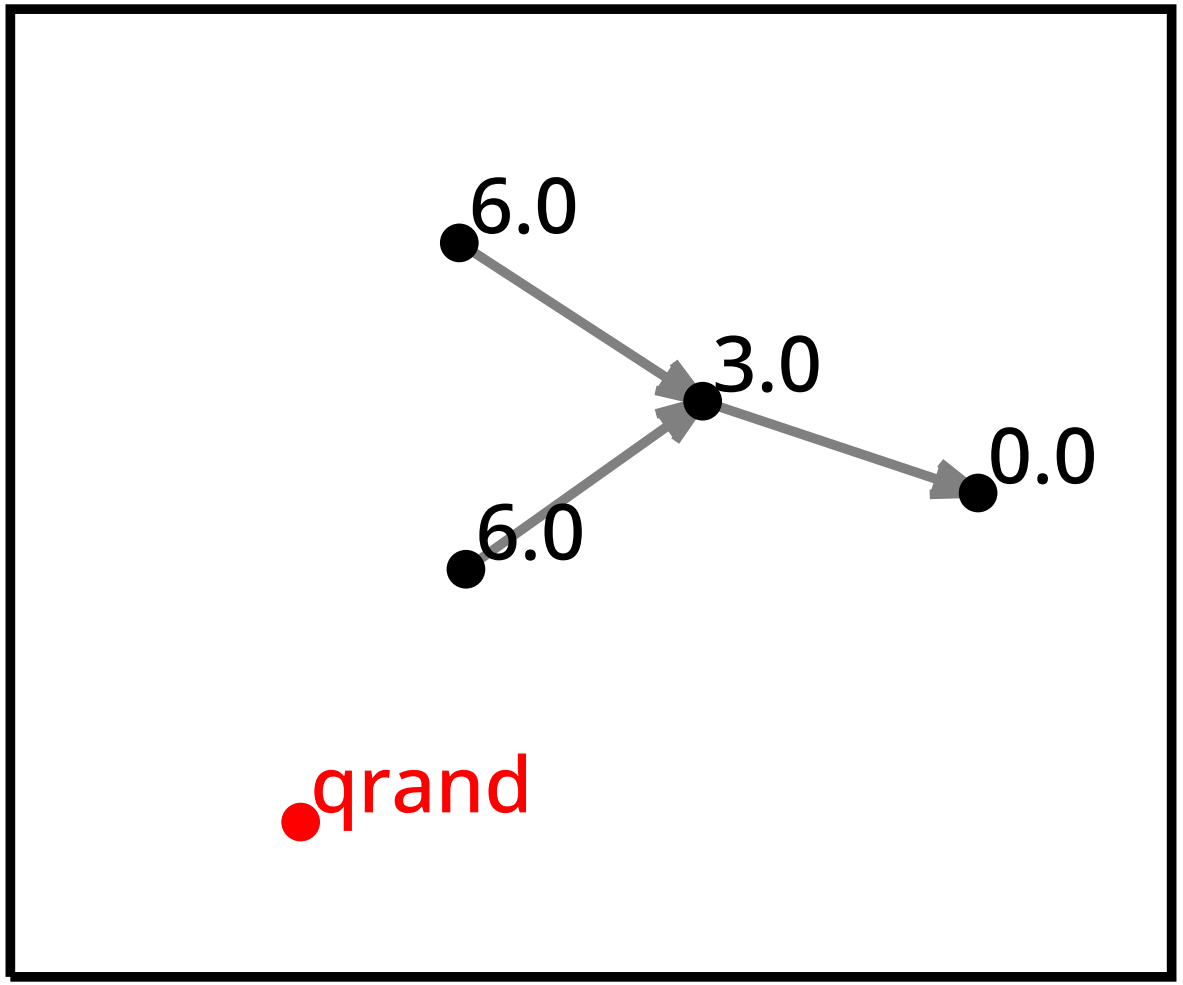
RRT*: Example 1

Rewire if needed; 0 changes this time.



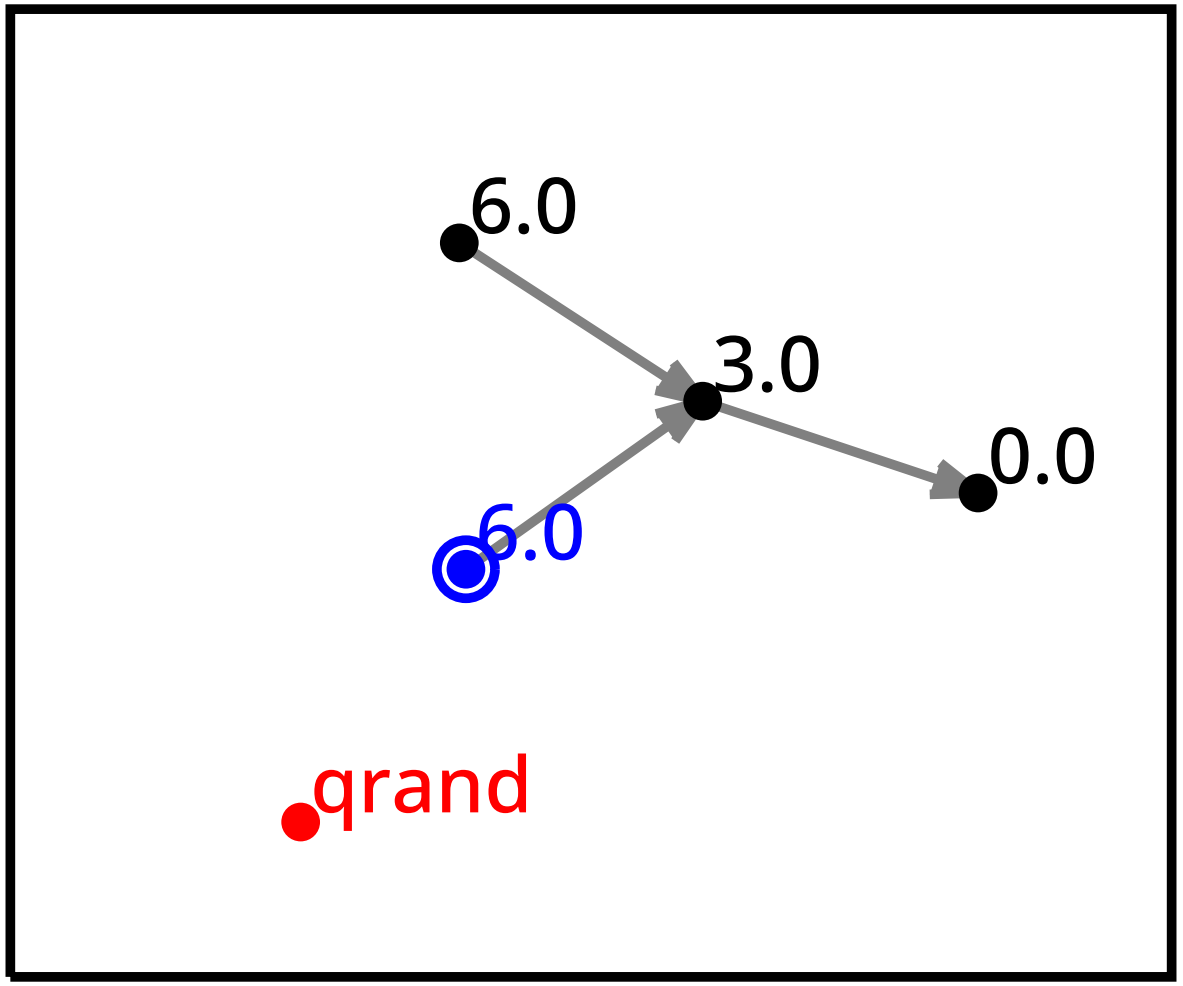
RRT*: Example 1

Choose a random sample.



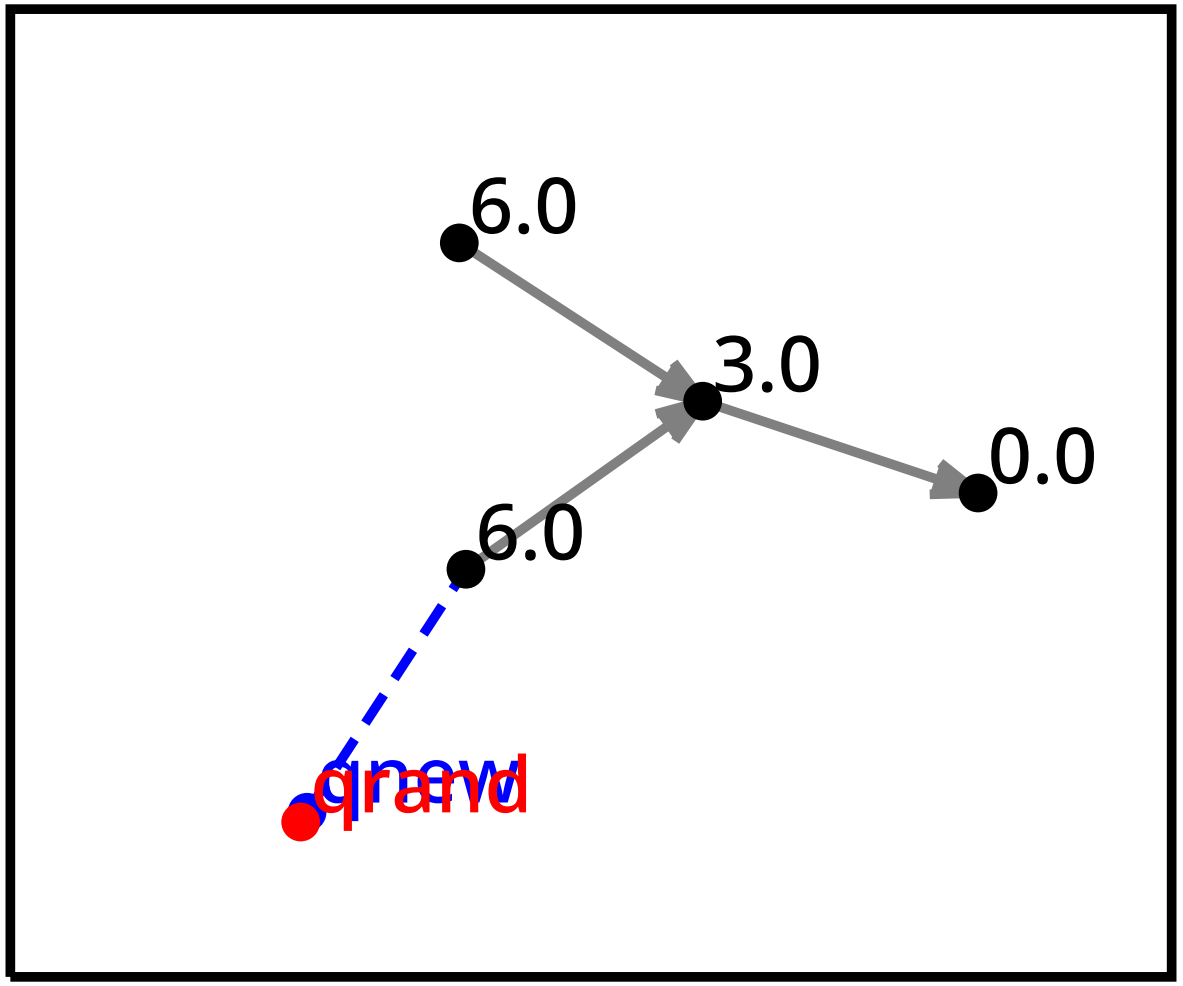
RRT*: Example 1

Find the nearest neighbor.



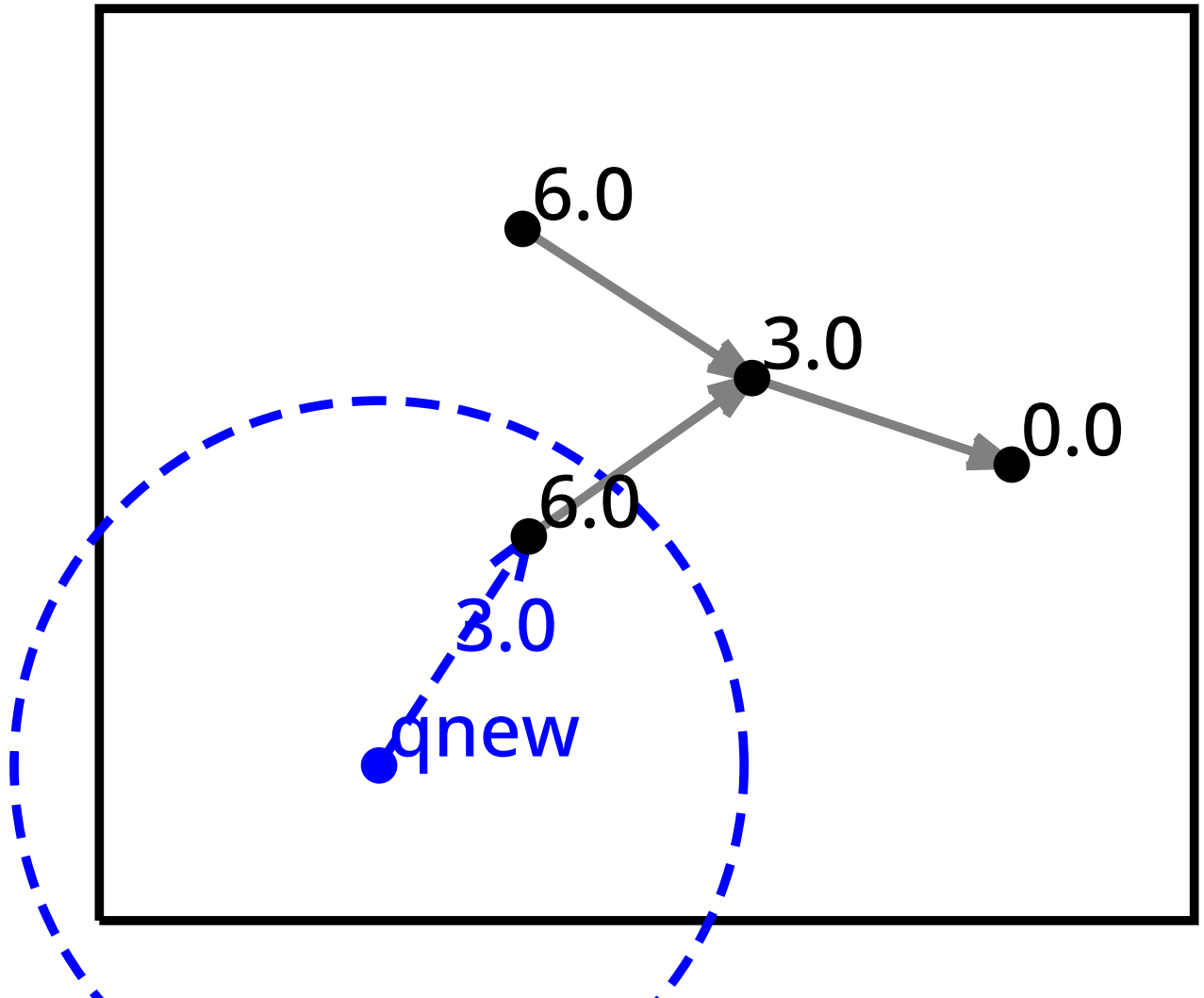
RRT*: Example 1

Extend from the nearest neighbor toward the sample.



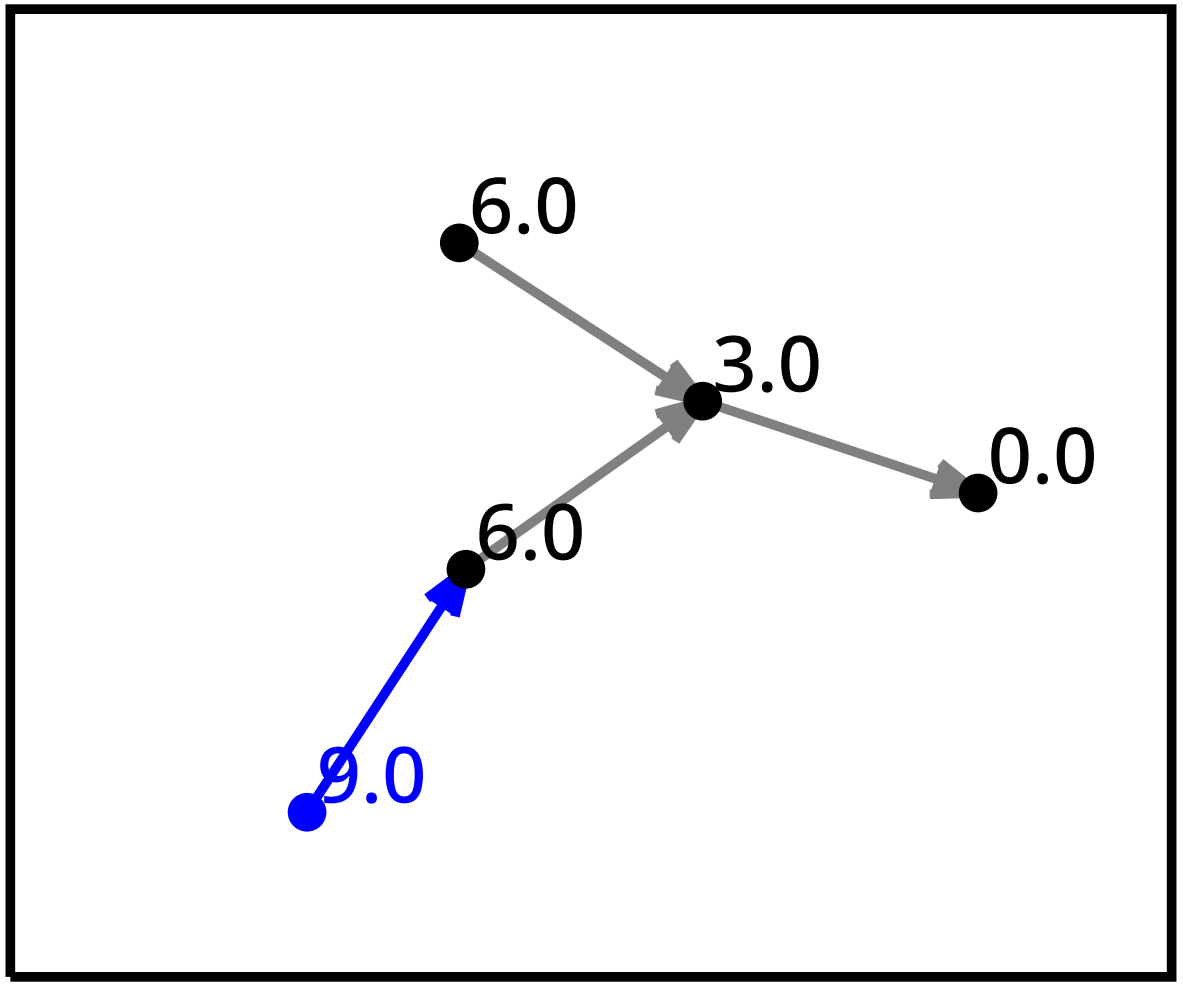
RRT*: Example 1

Find candidate parents for the new node; 1 candidate this time.



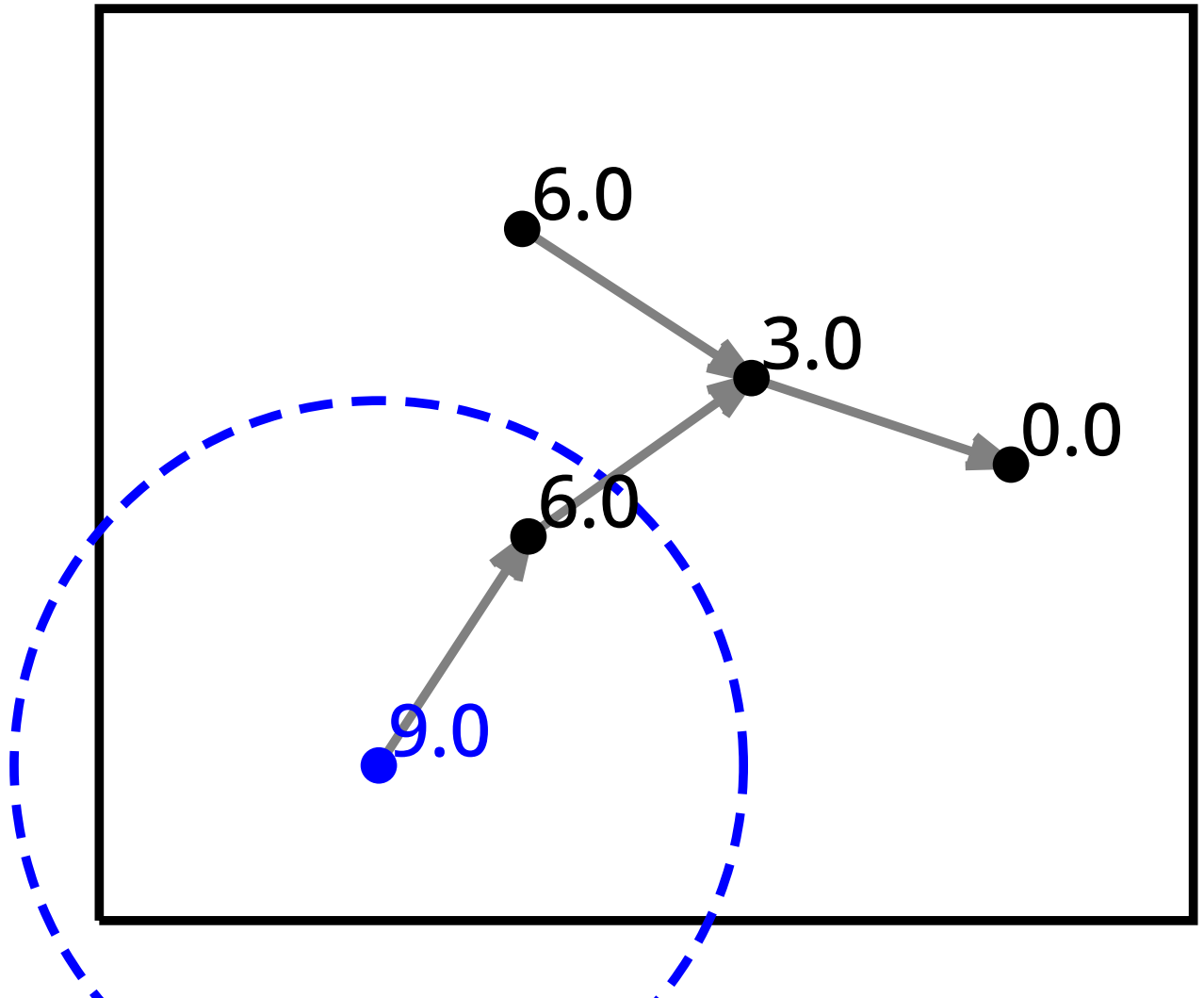
RRT*: Example 1

Choose the best parent for the new node.



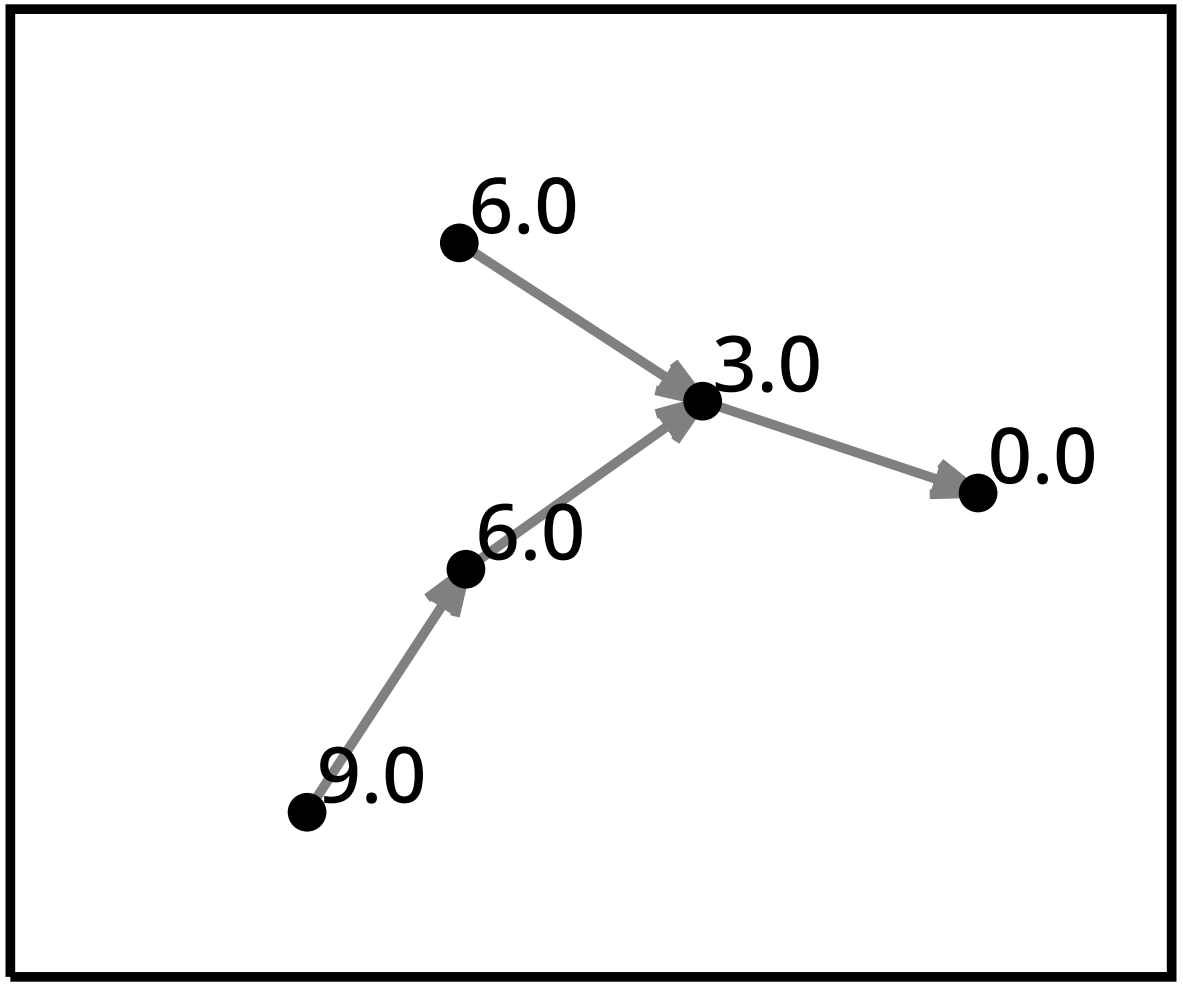
RRT*: Example 1

Find rewiring candidates near the new node; 0 candidates this time.



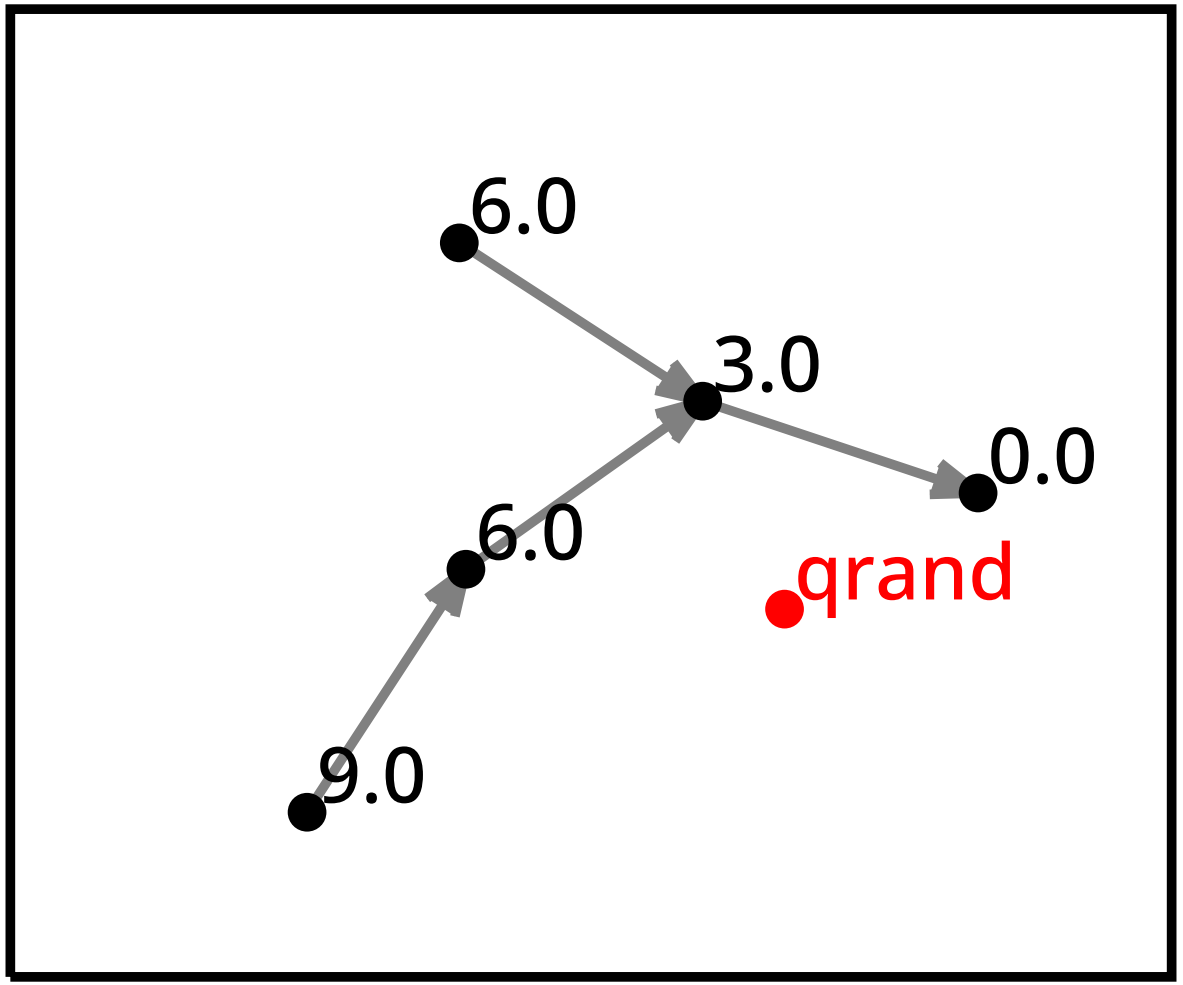
RRT*: Example 1

Rewire if needed; 0 changes this time.



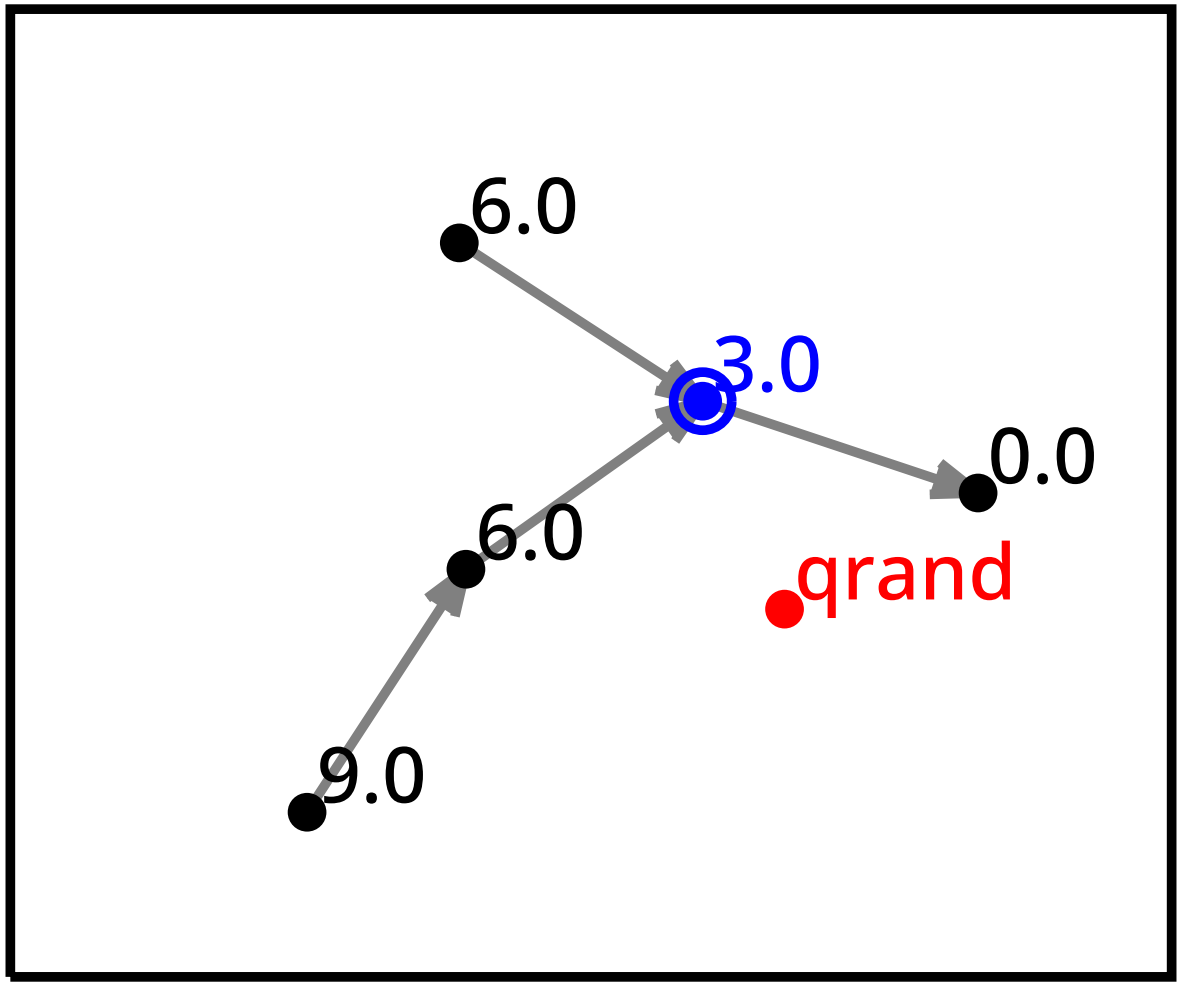
RRT*: Example 1

Choose a random sample.



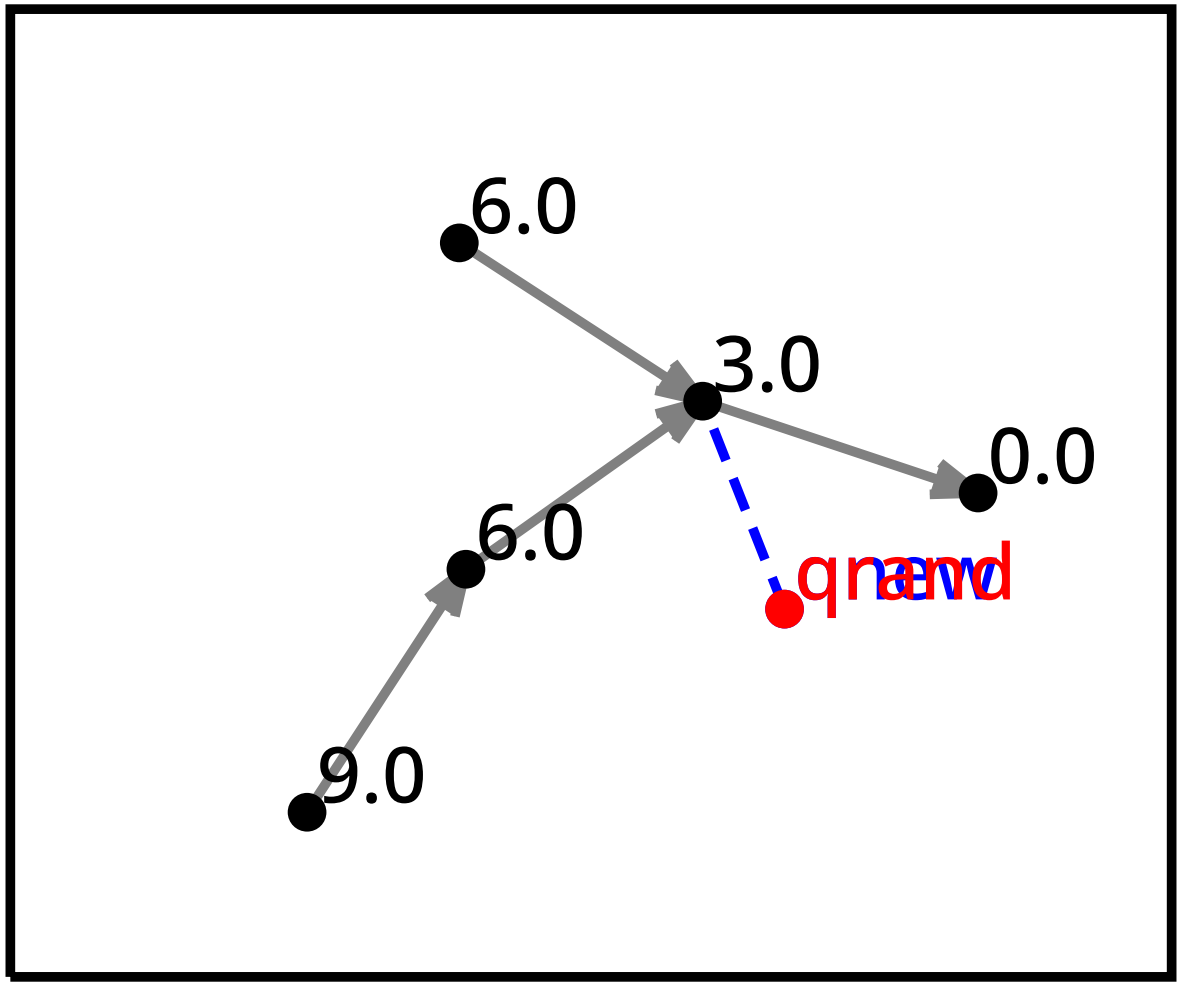
RRT*: Example 1

Find the nearest neighbor.



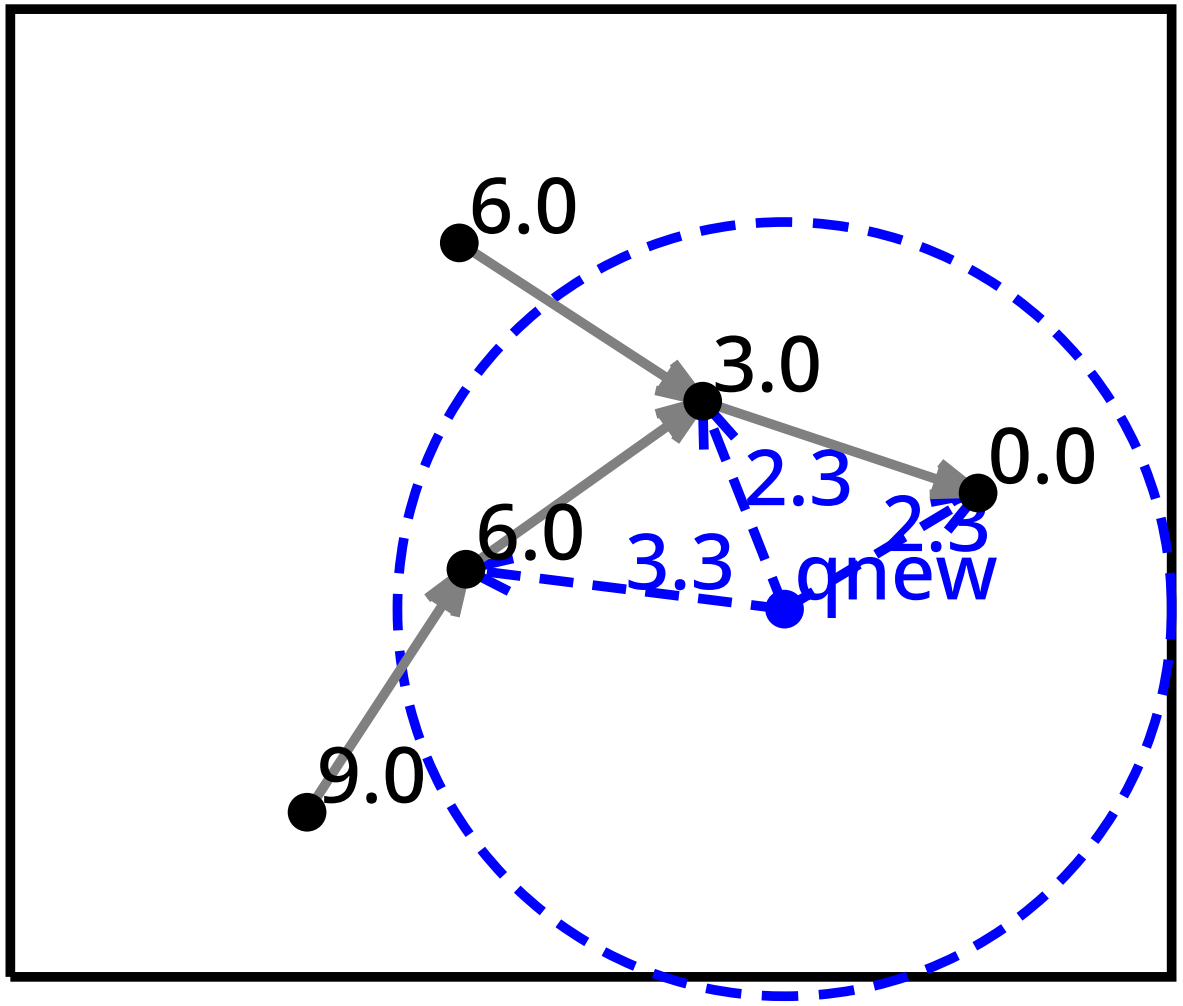
RRT*: Example 1

Extend from the nearest neighbor toward the sample.



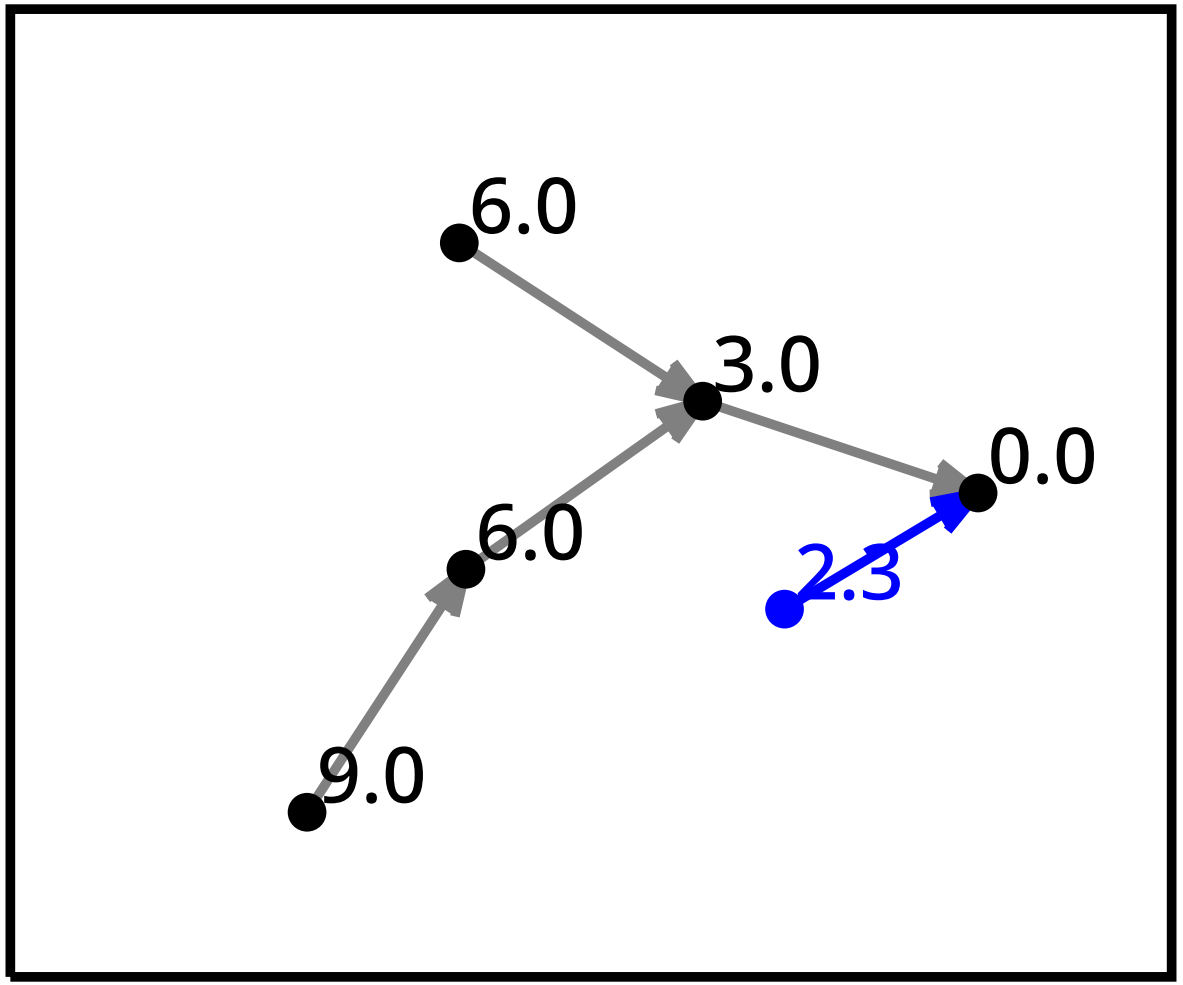
RRT*: Example 1

Find candidate parents for the new node; 3 candidates this time.



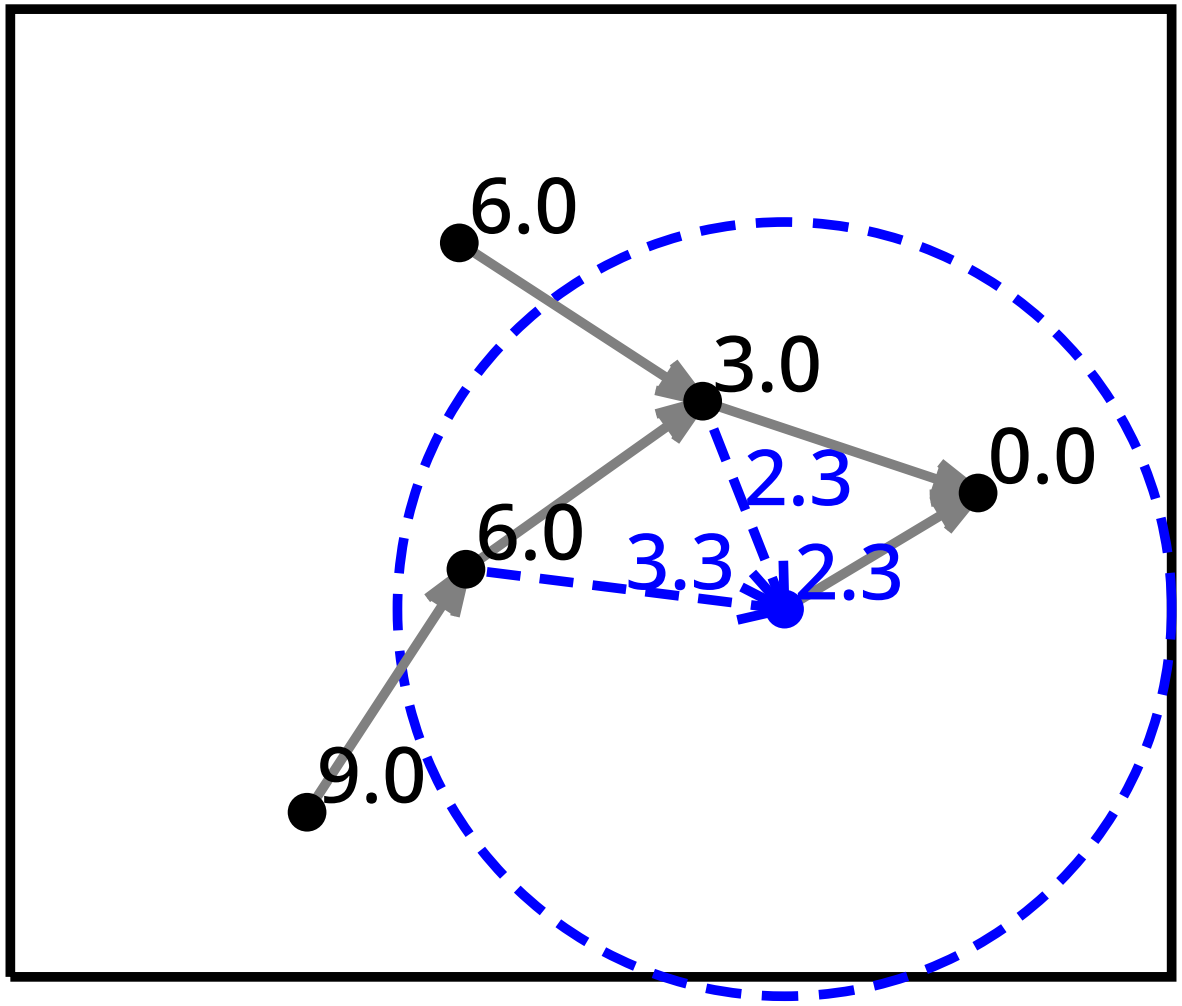
RRT*: Example 1

Choose the best parent for the new node.



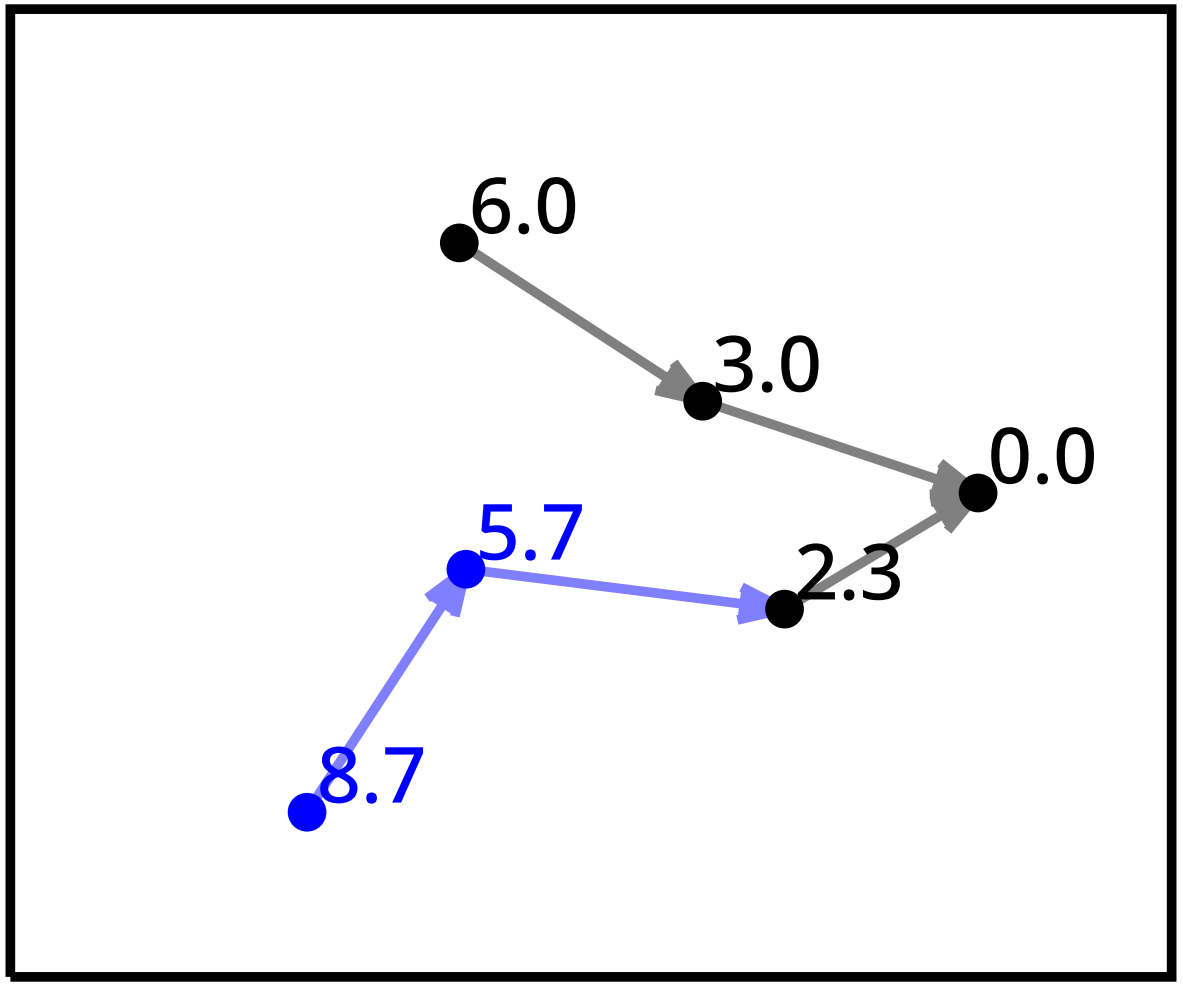
RRT*: Example 1

Find rewiring candidates near the new node; 2 candidates this time.



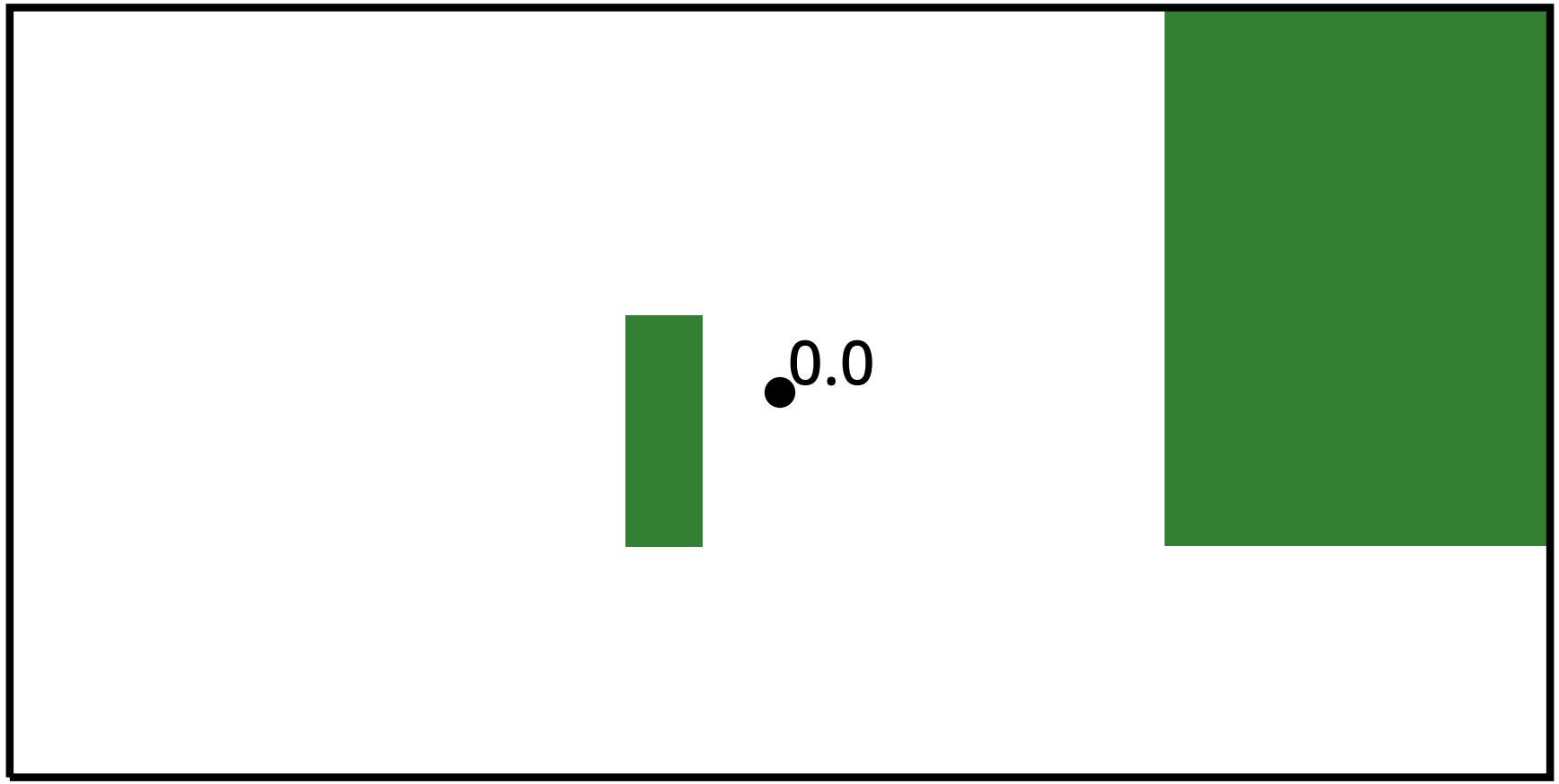
RRT*: Example 1

Rewire if needed; 2 changes this time.



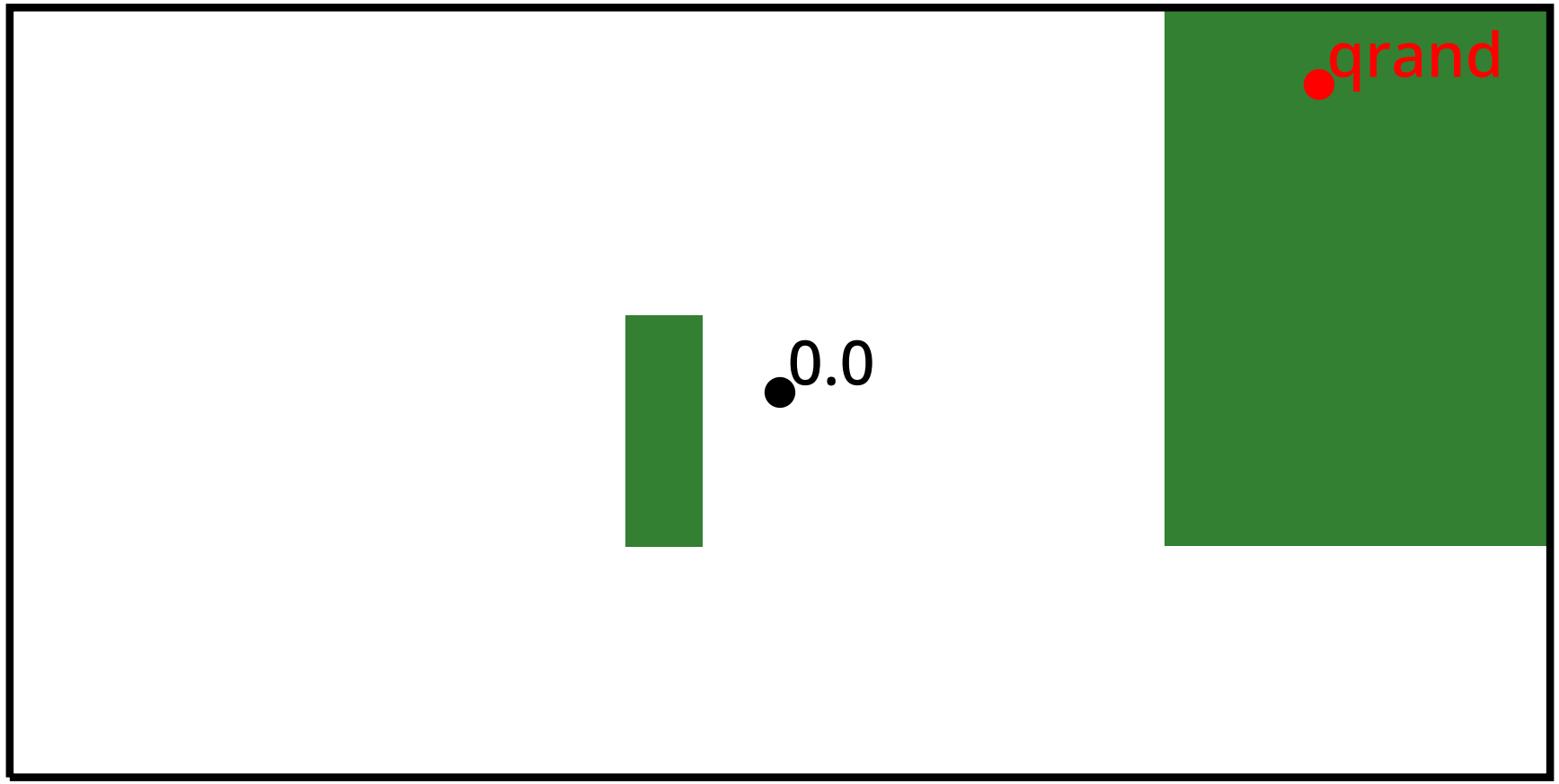
RRT*: Example 2

Start from a tree with just the start configuration.



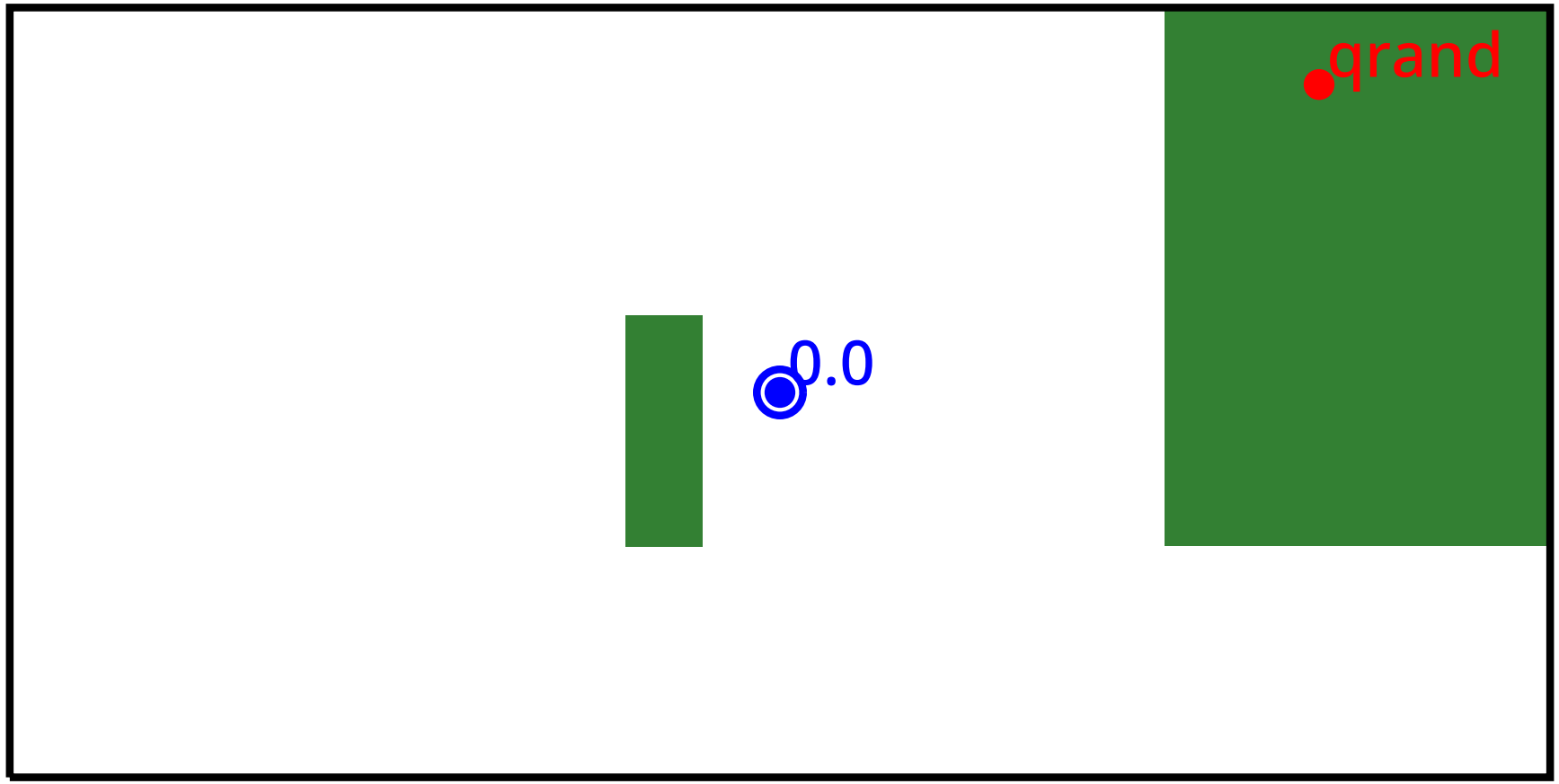
RRT*: Example 2

Choose a random sample.



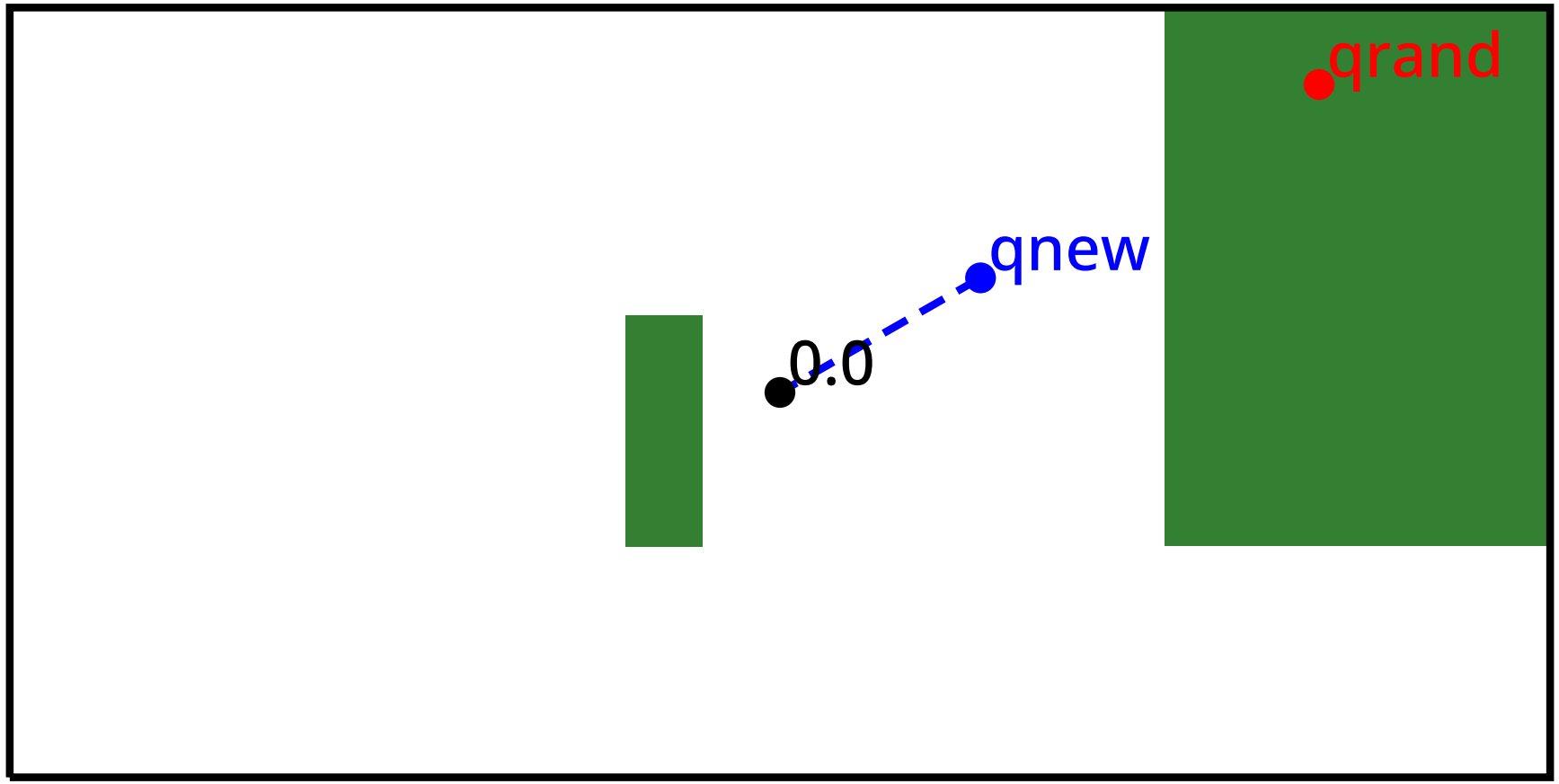
RRT*: Example 2

Find the nearest neighbor.



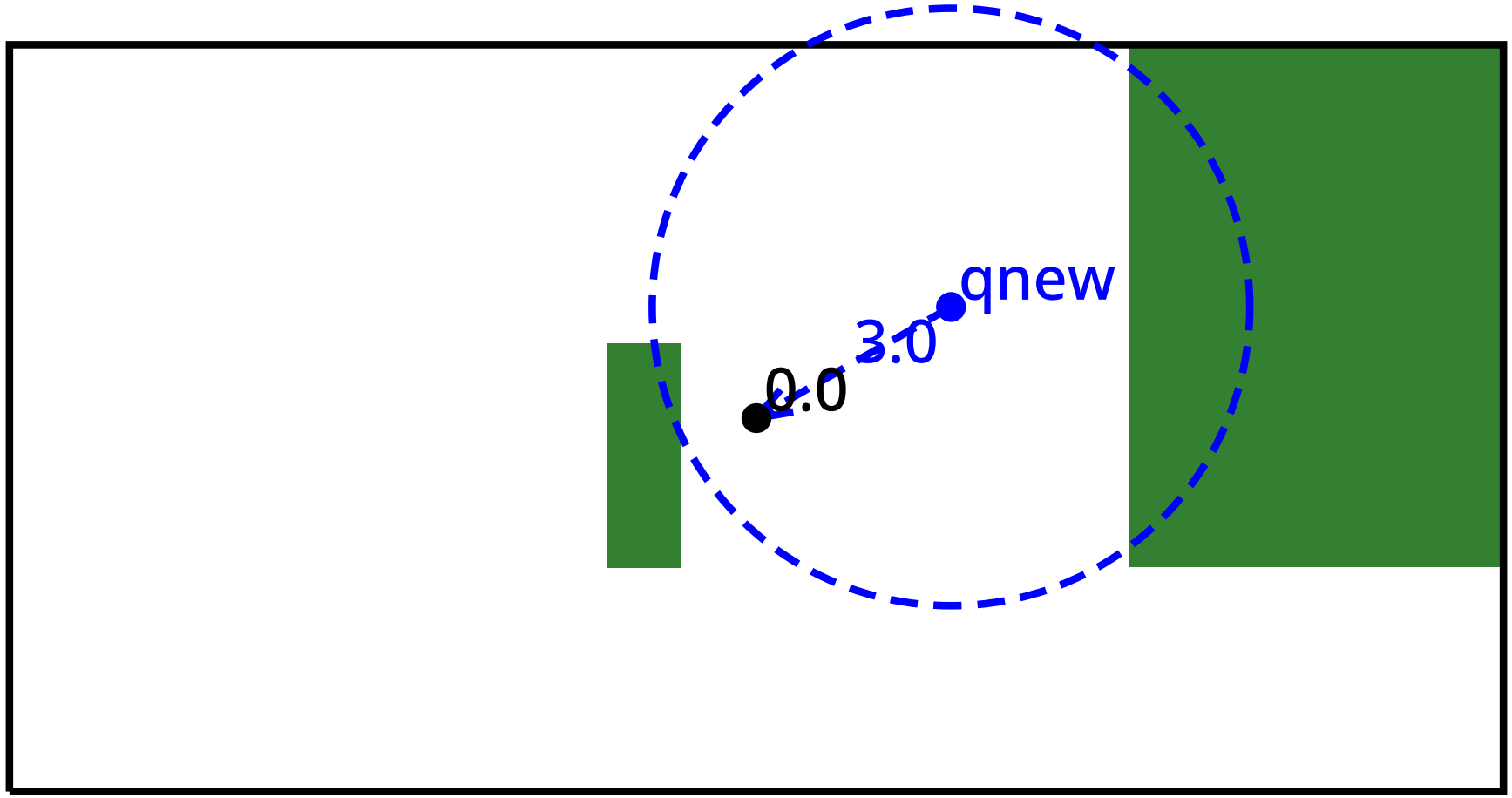
RRT*: Example 2

Extend from the nearest neighbor toward the sample.



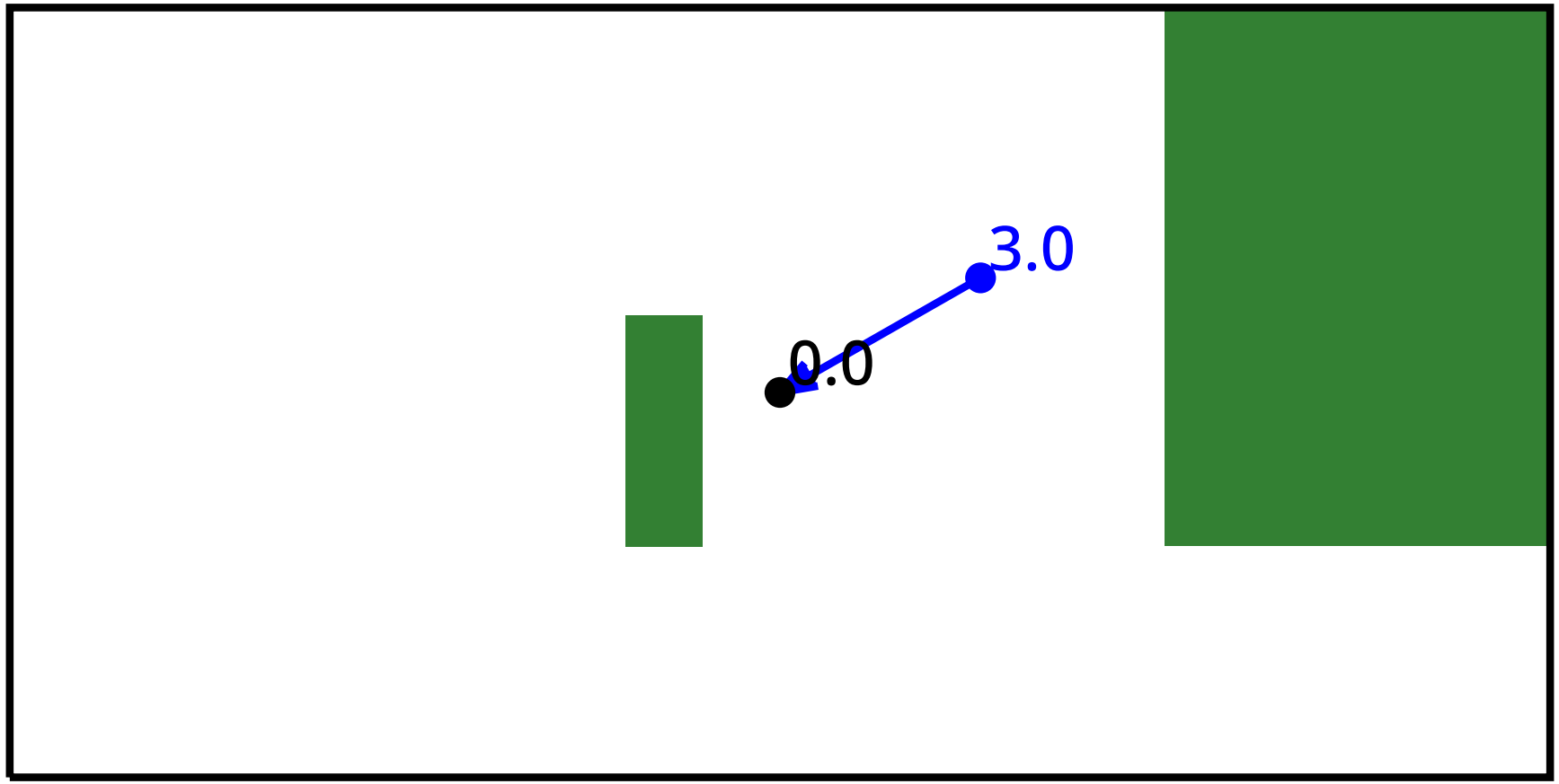
RRT*: Example 2

Find candidate parents for the new node; 1 candidate this time.



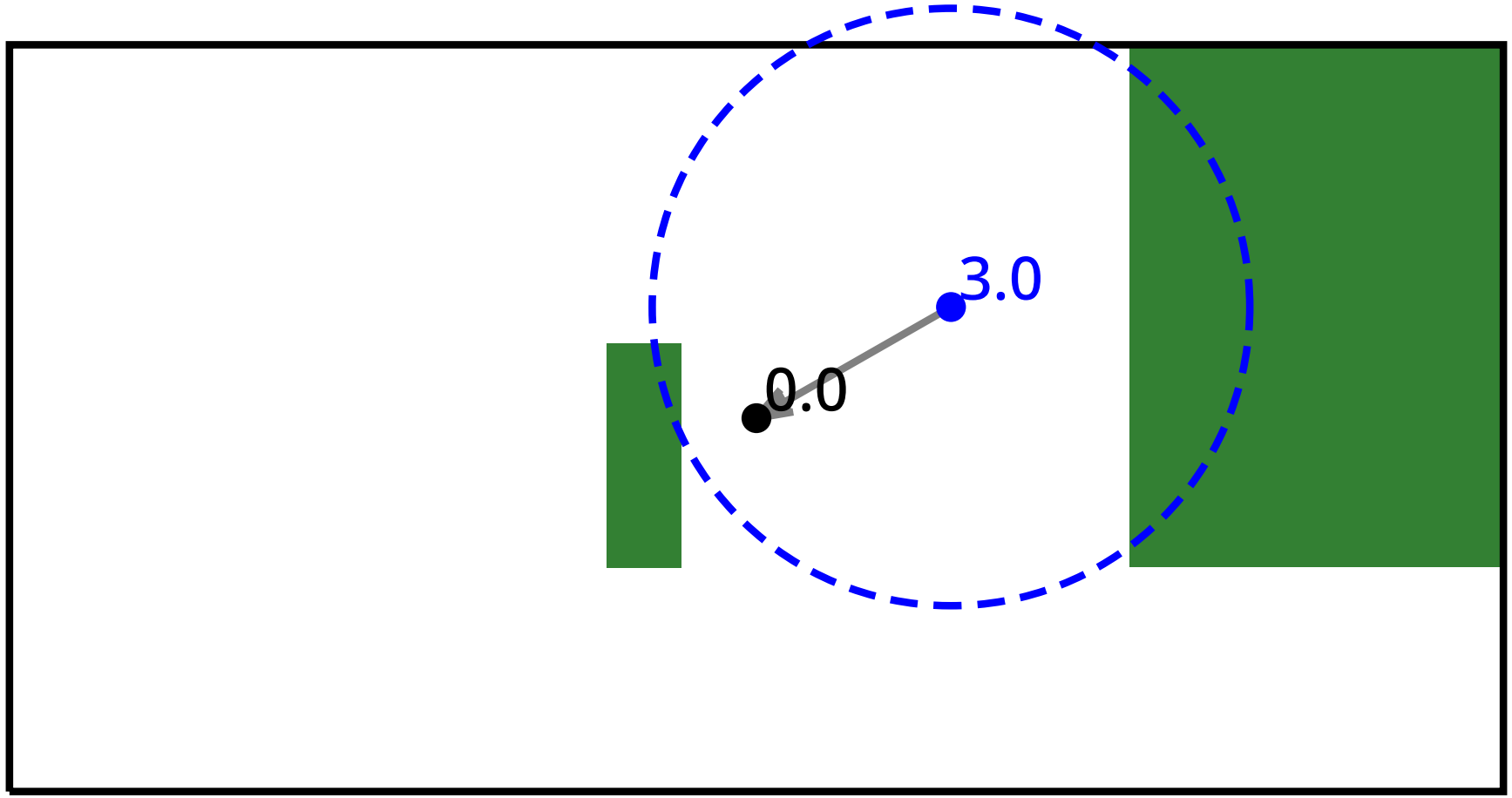
RRT*: Example 2

Choose the best parent for the new node.



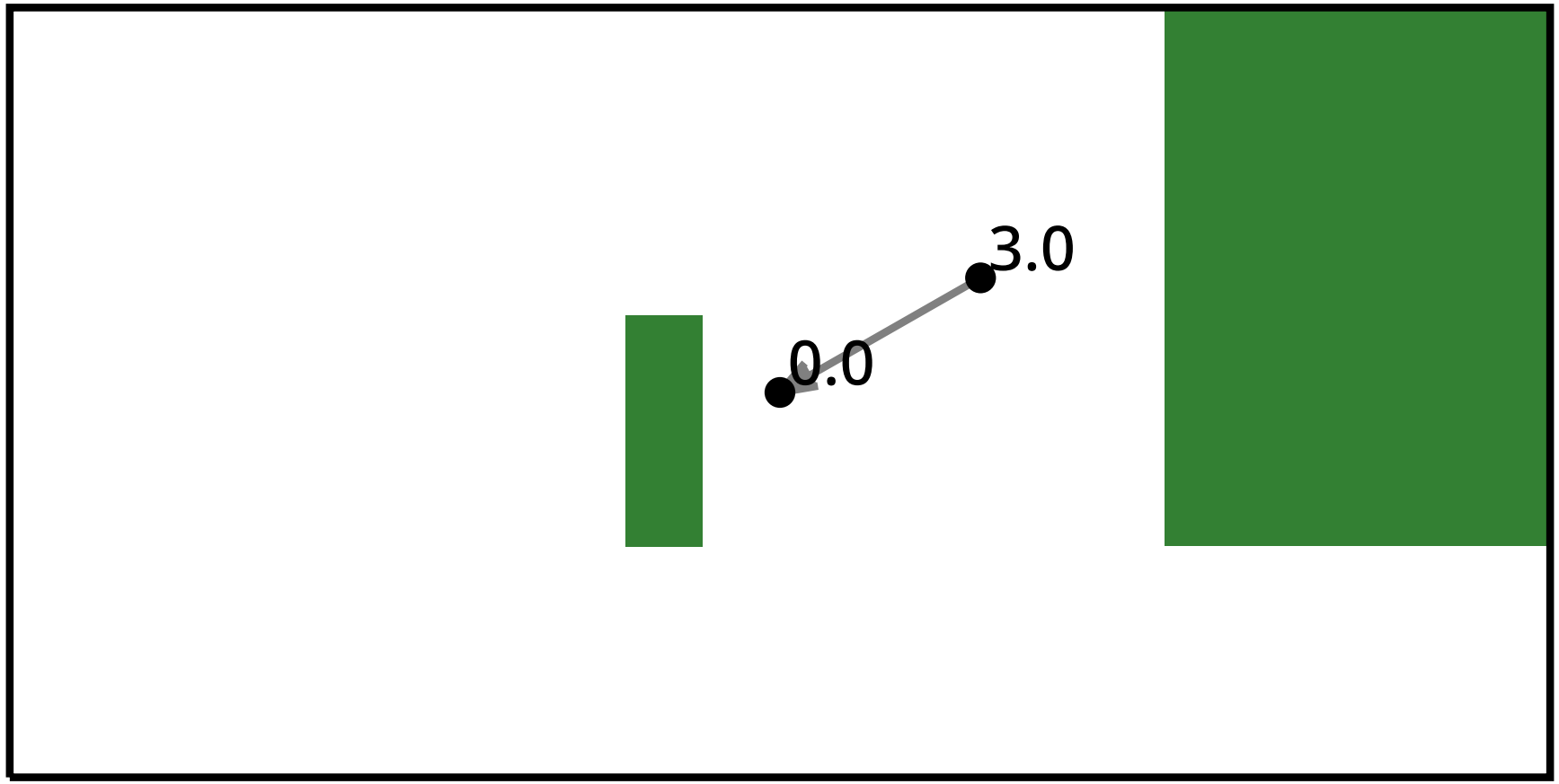
RRT*: Example 2

Find rewiring candidates near the new node; 0 candidates this time.



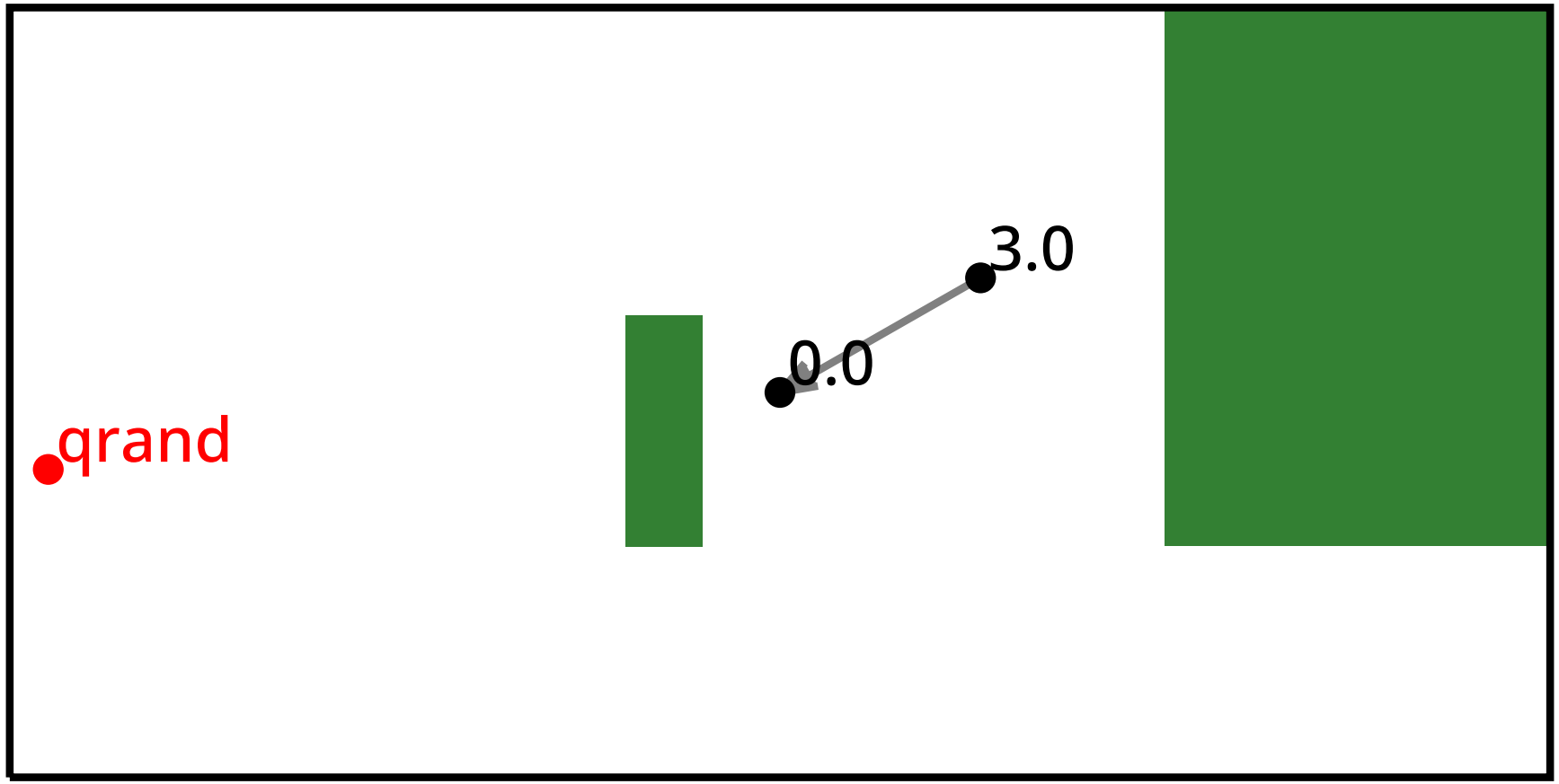
RRT*: Example 2

Rewire if needed; 0 changes this time.



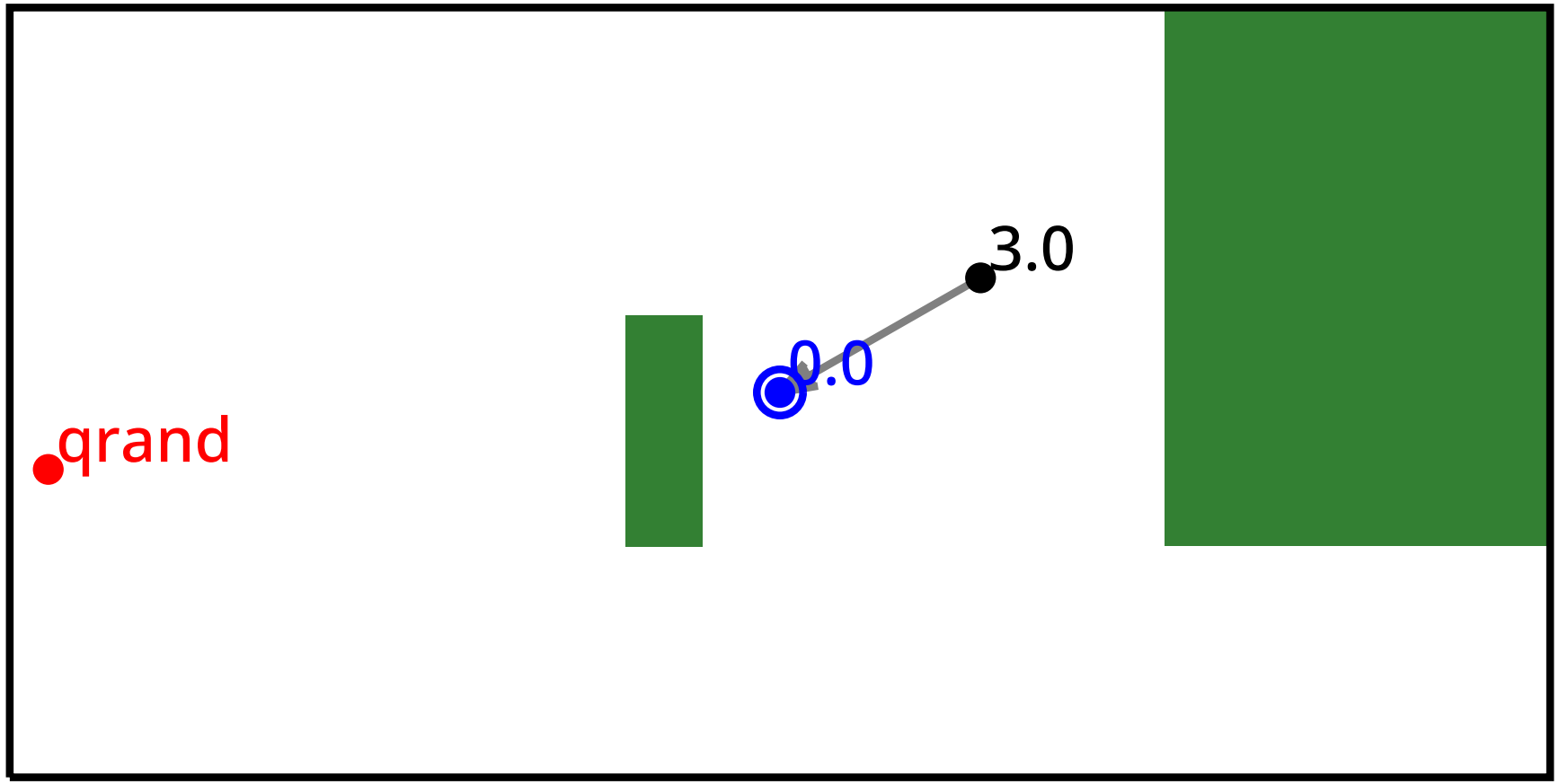
RRT*: Example 2

Choose a random sample.



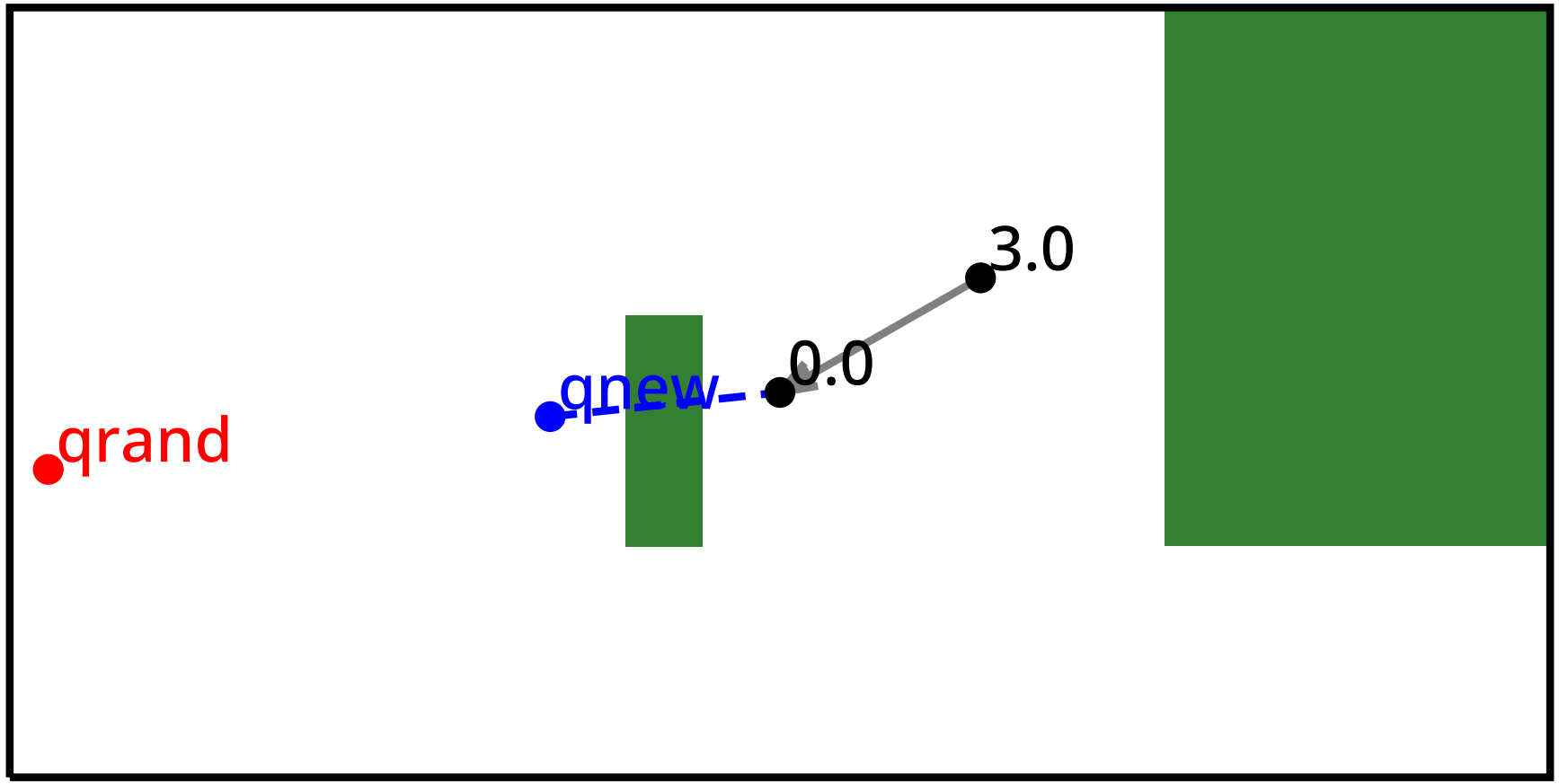
RRT*: Example 2

Find the nearest neighbor.



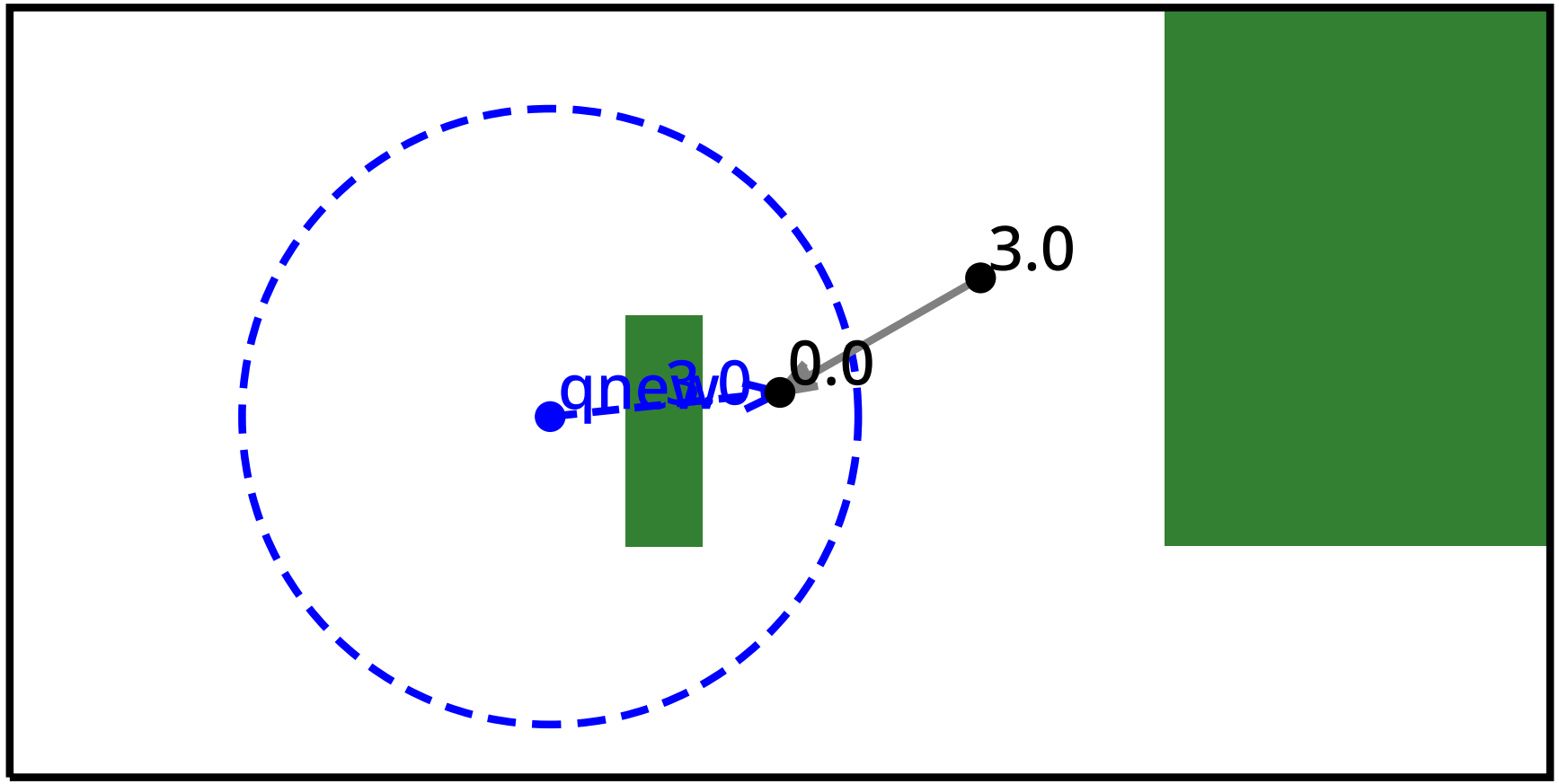
RRT*: Example 2

Extend from the nearest neighbor toward the sample.



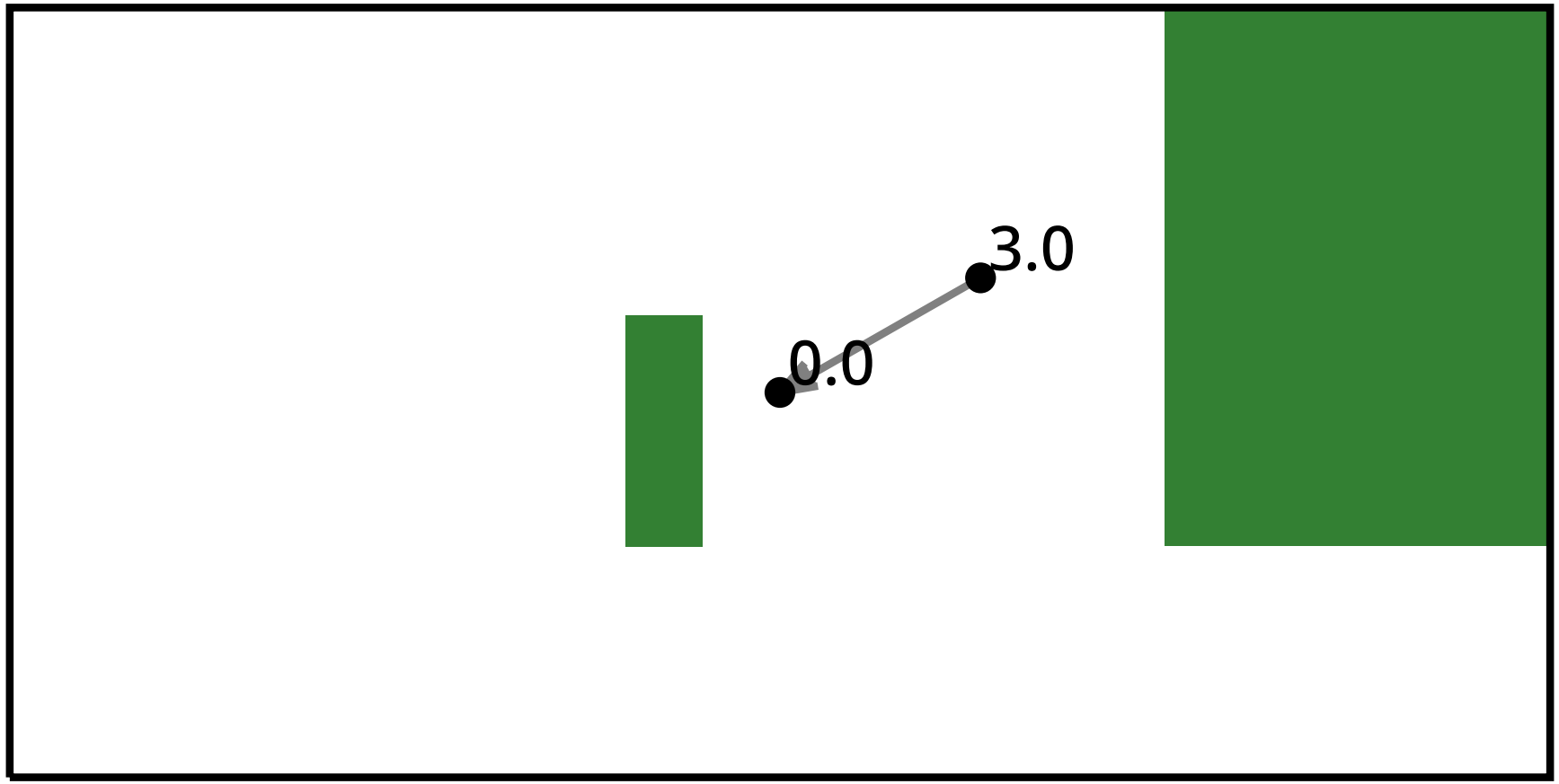
RRT*: Example 2

Find candidate parents for the new node; 1 candidate this time.



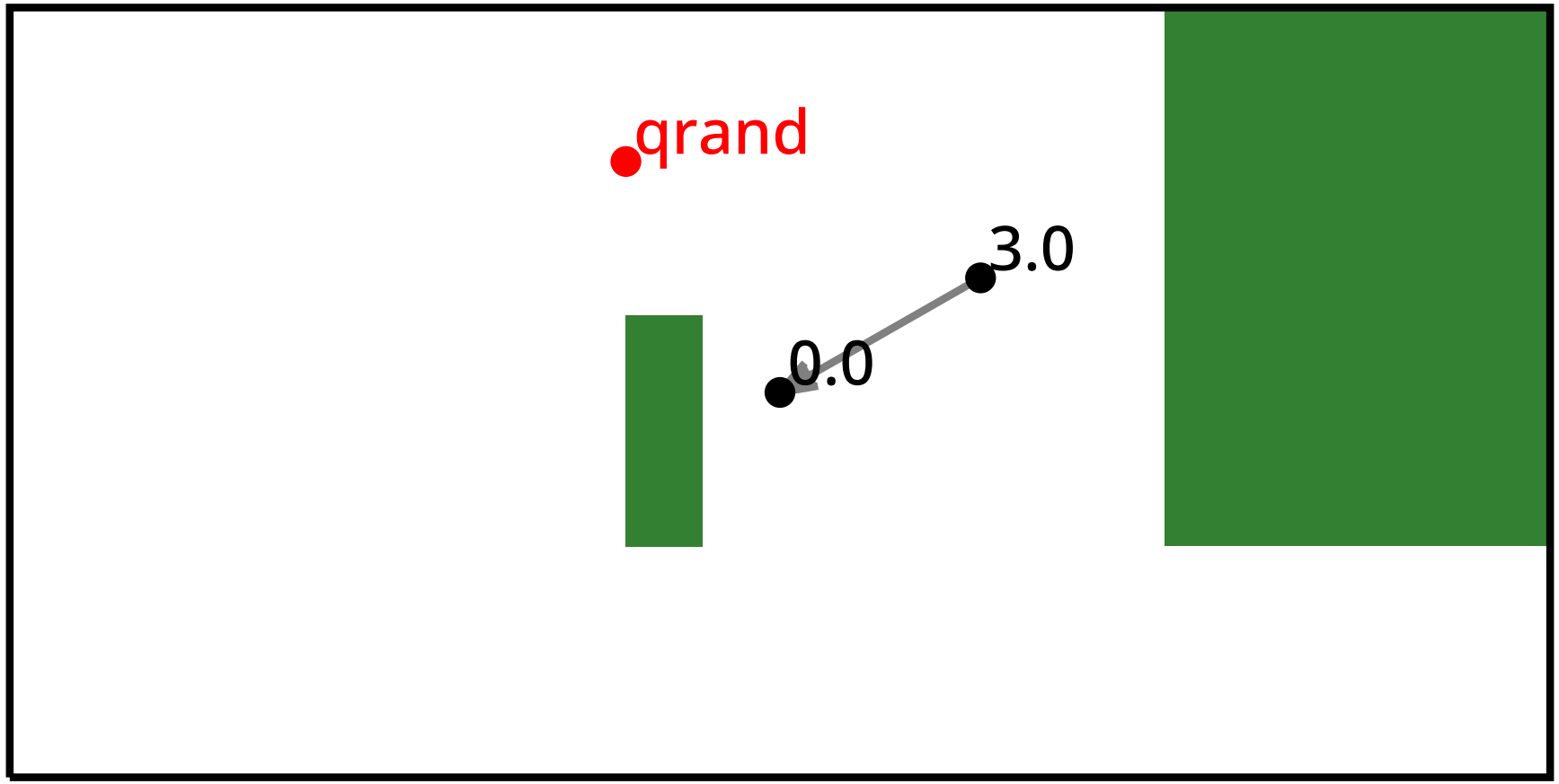
RRT*: Example 2

No valid parent available for the new node.



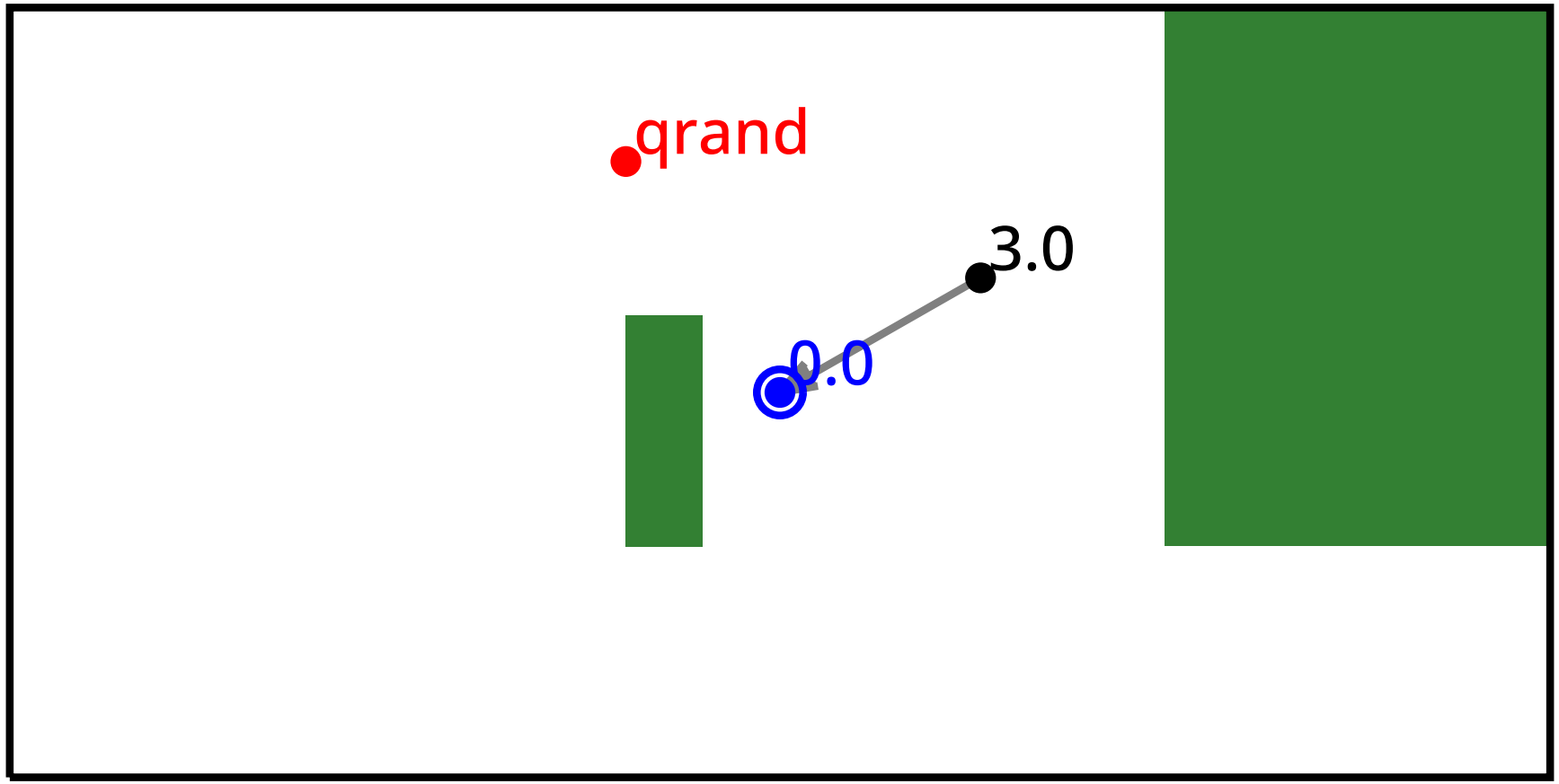
RRT*: Example 2

Choose a random sample.



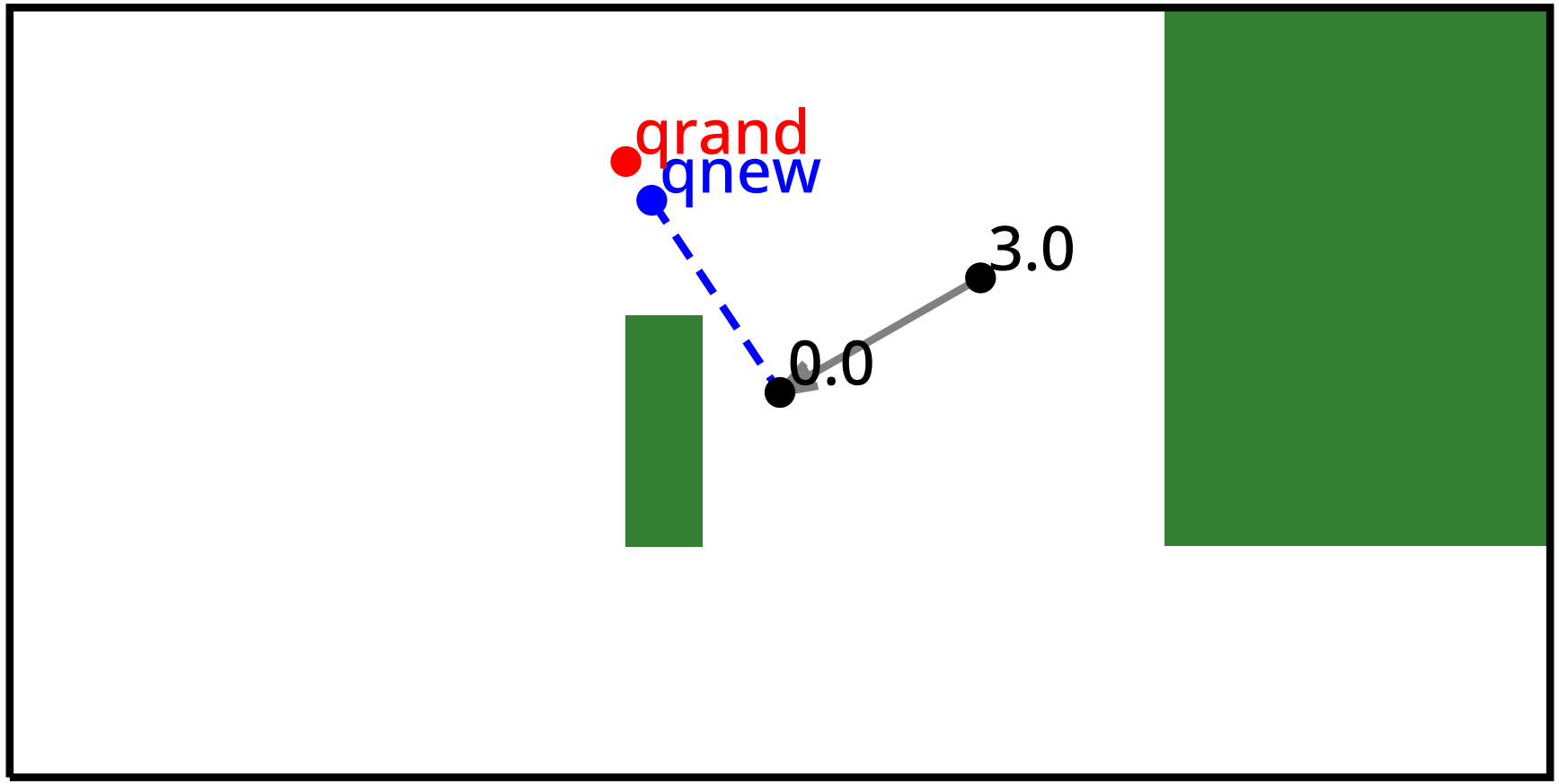
RRT*: Example 2

Find the nearest neighbor.



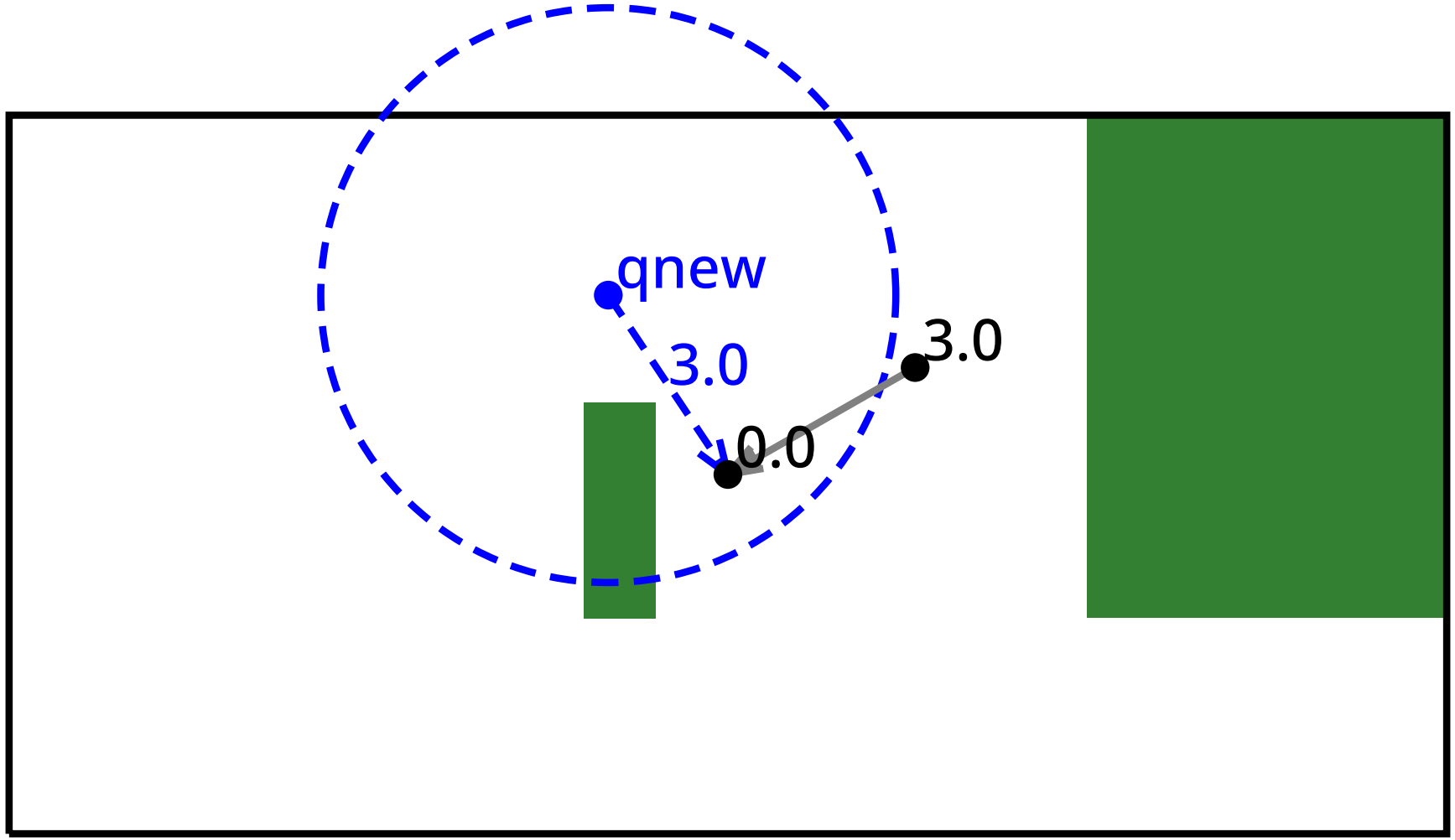
RRT*: Example 2

Extend from the nearest neighbor toward the sample.



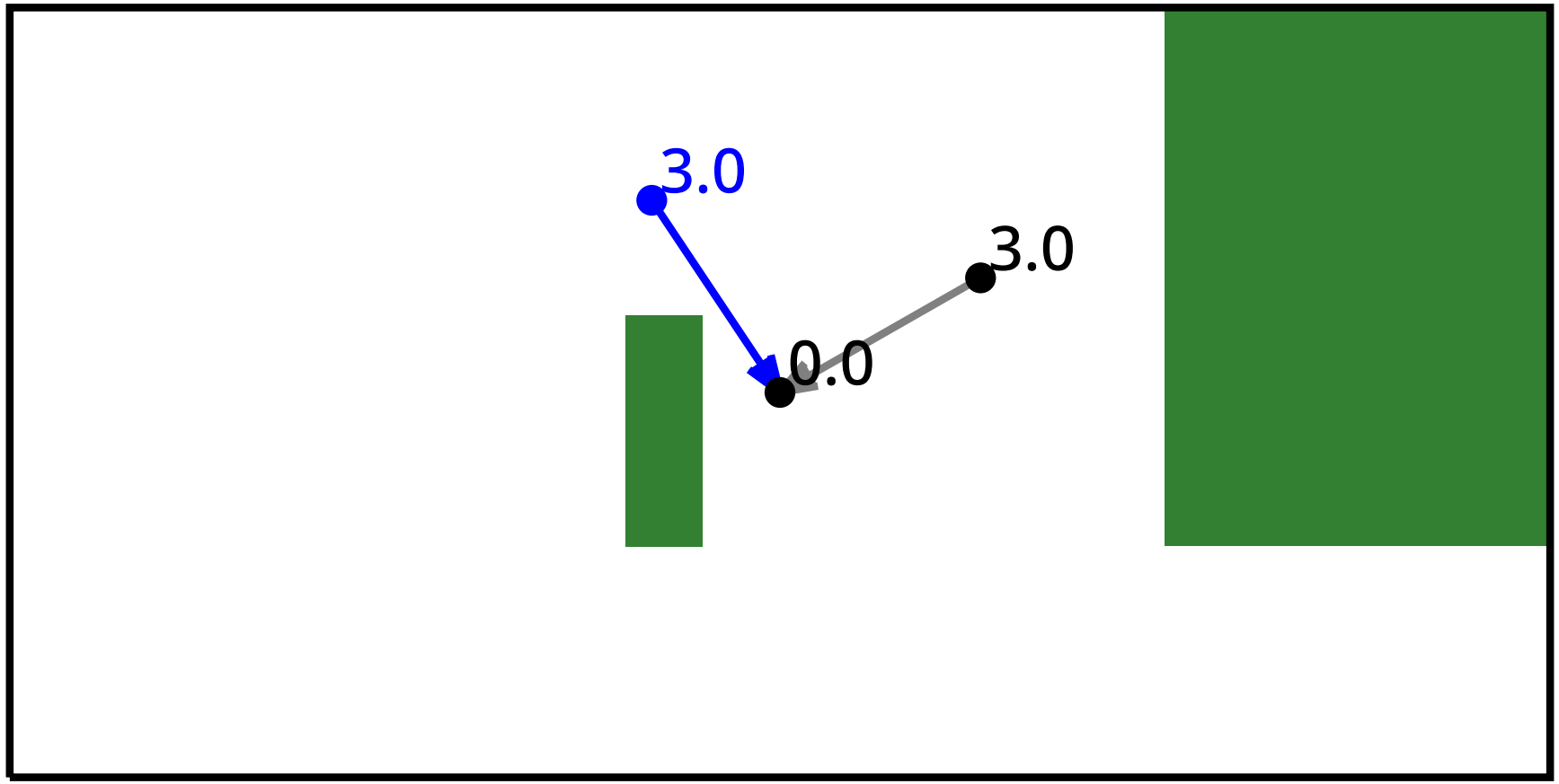
RRT*: Example 2

Find candidate parents for the new node; 1 candidate this time.



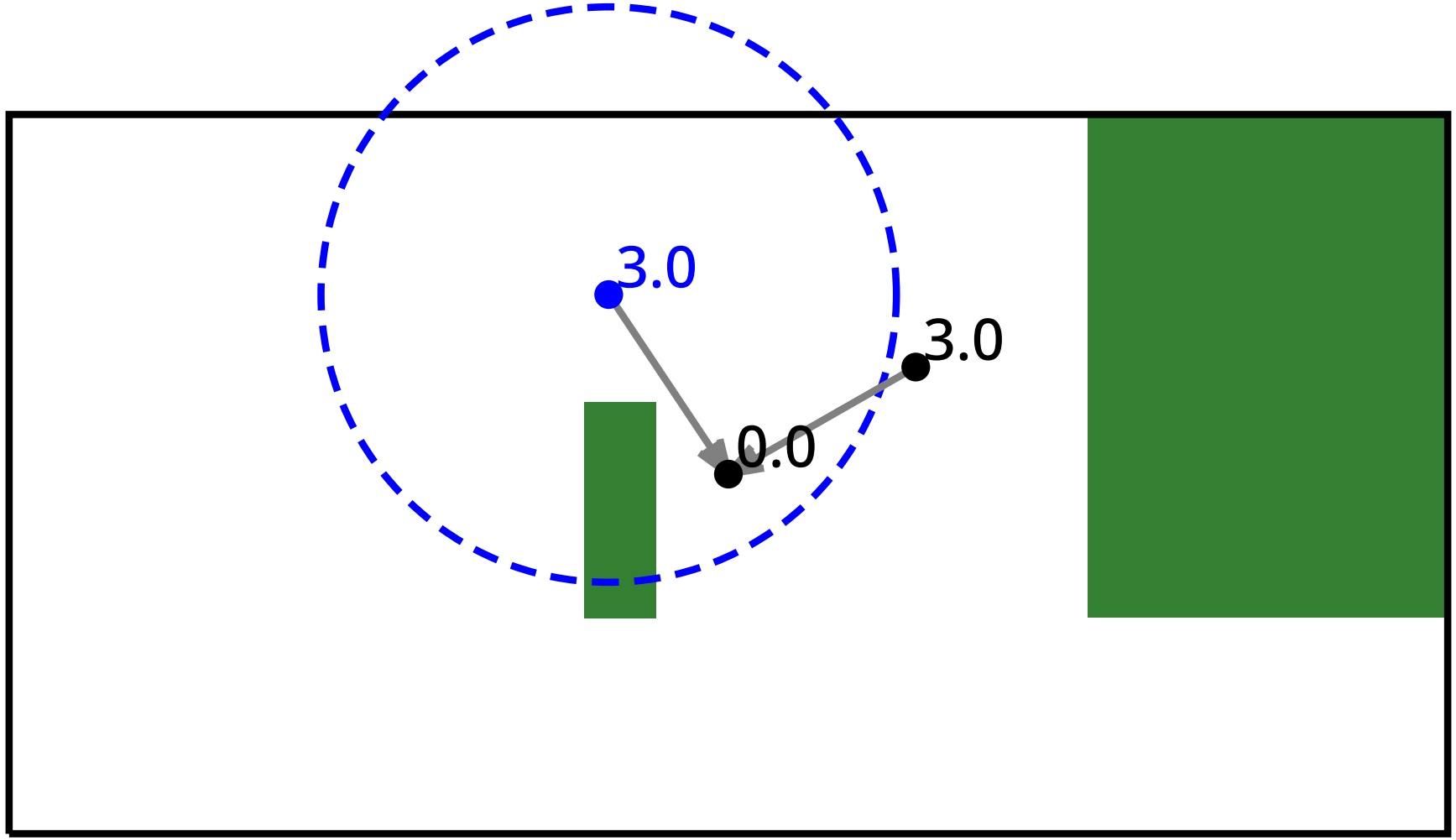
RRT*: Example 2

Choose the best parent for the new node.



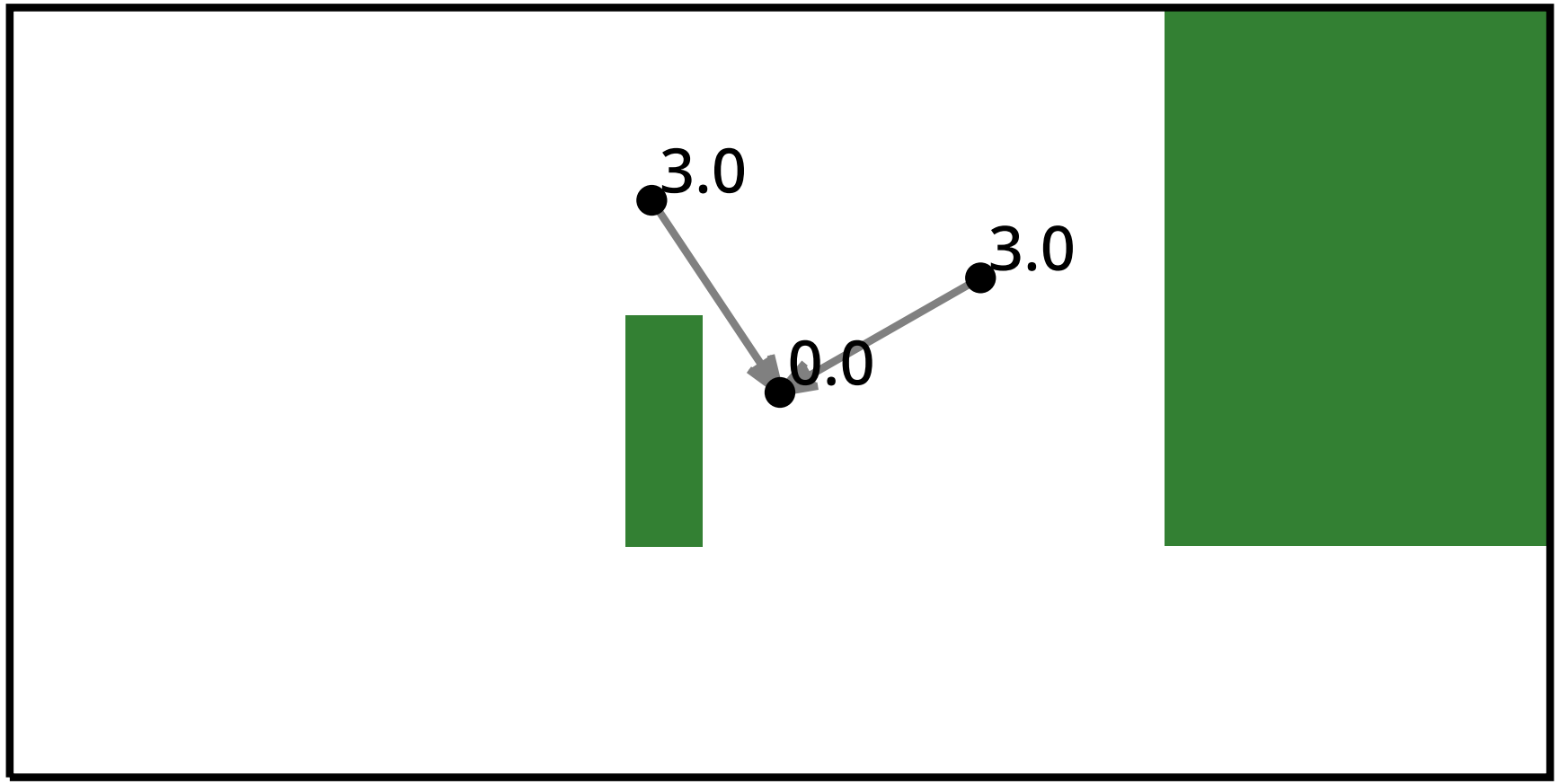
RRT*: Example 2

Find rewiring candidates near the new node; 0 candidates this time.



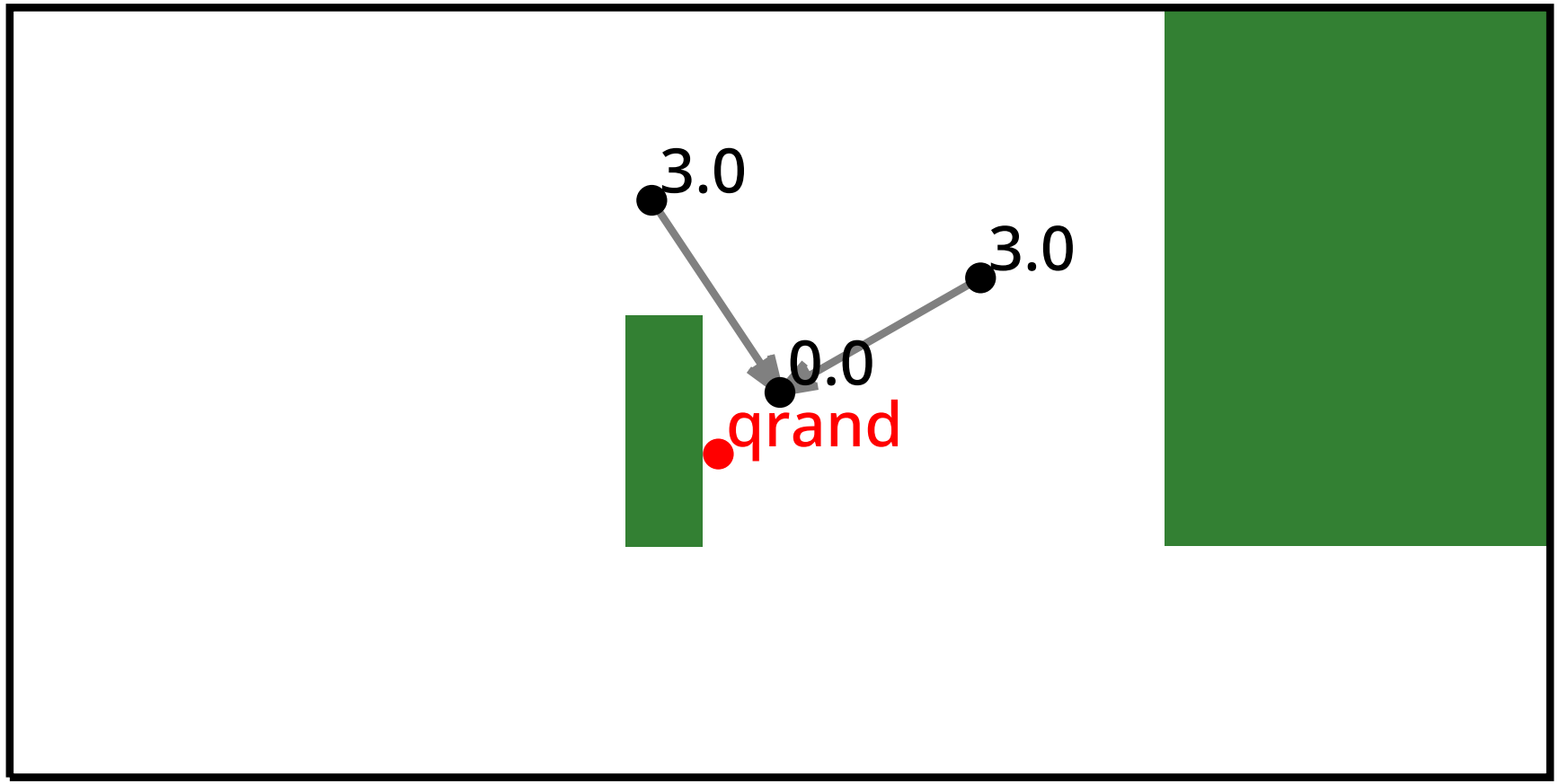
RRT*: Example 2

Rewire if needed; 0 changes this time.



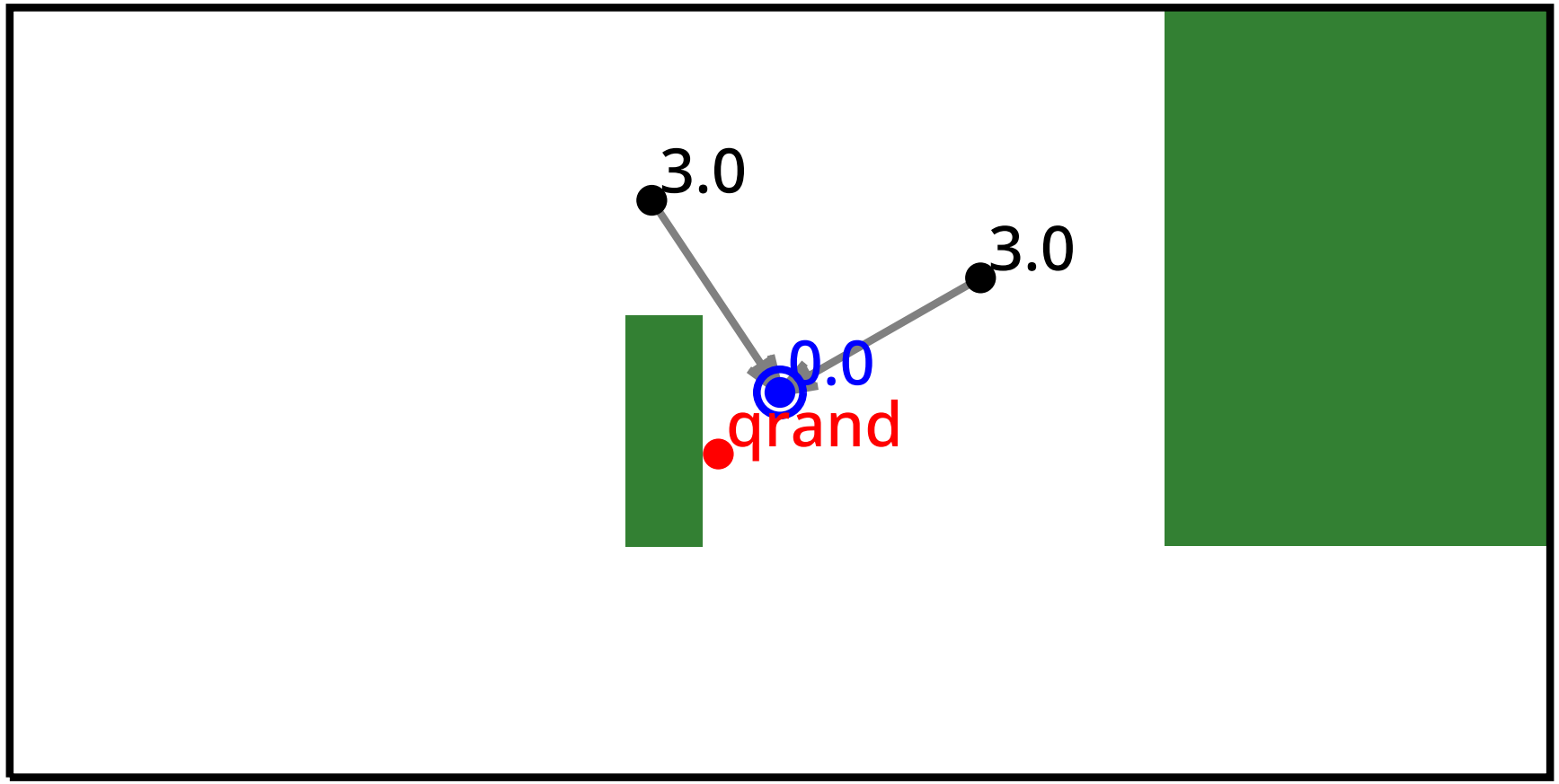
RRT*: Example 2

Choose a random sample.



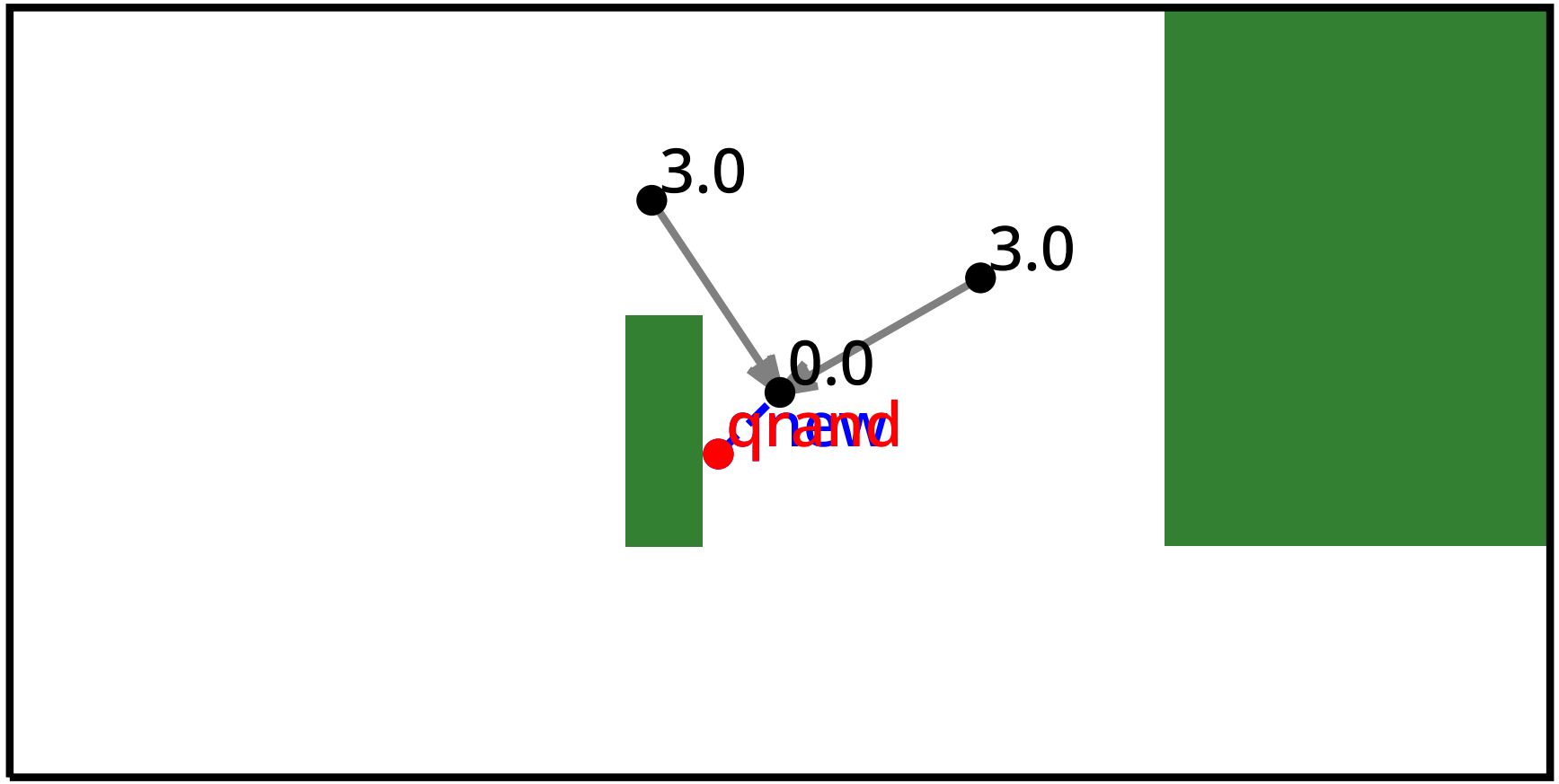
RRT*: Example 2

Find the nearest neighbor.



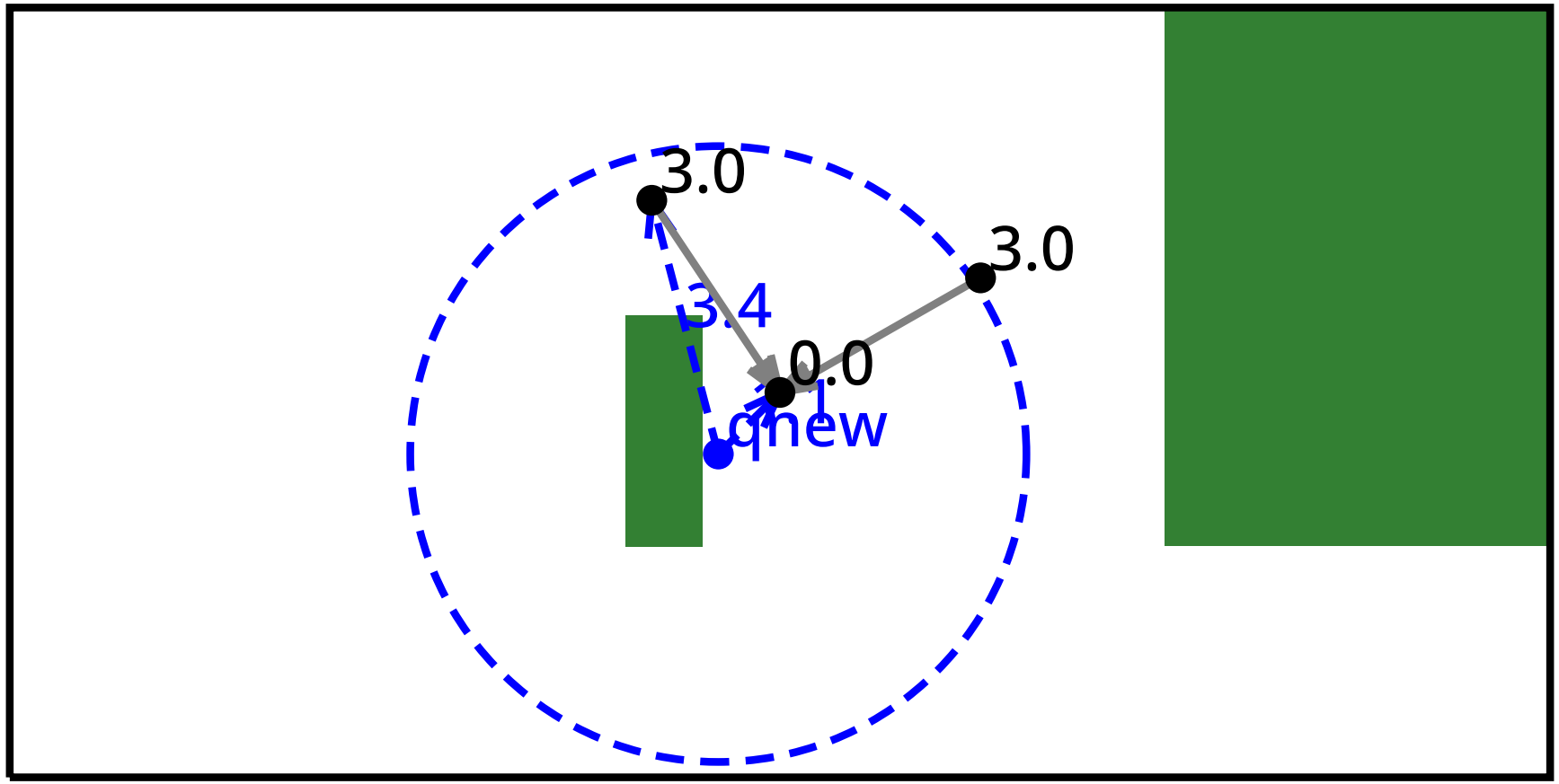
RRT*: Example 2

Extend from the nearest neighbor toward the sample.



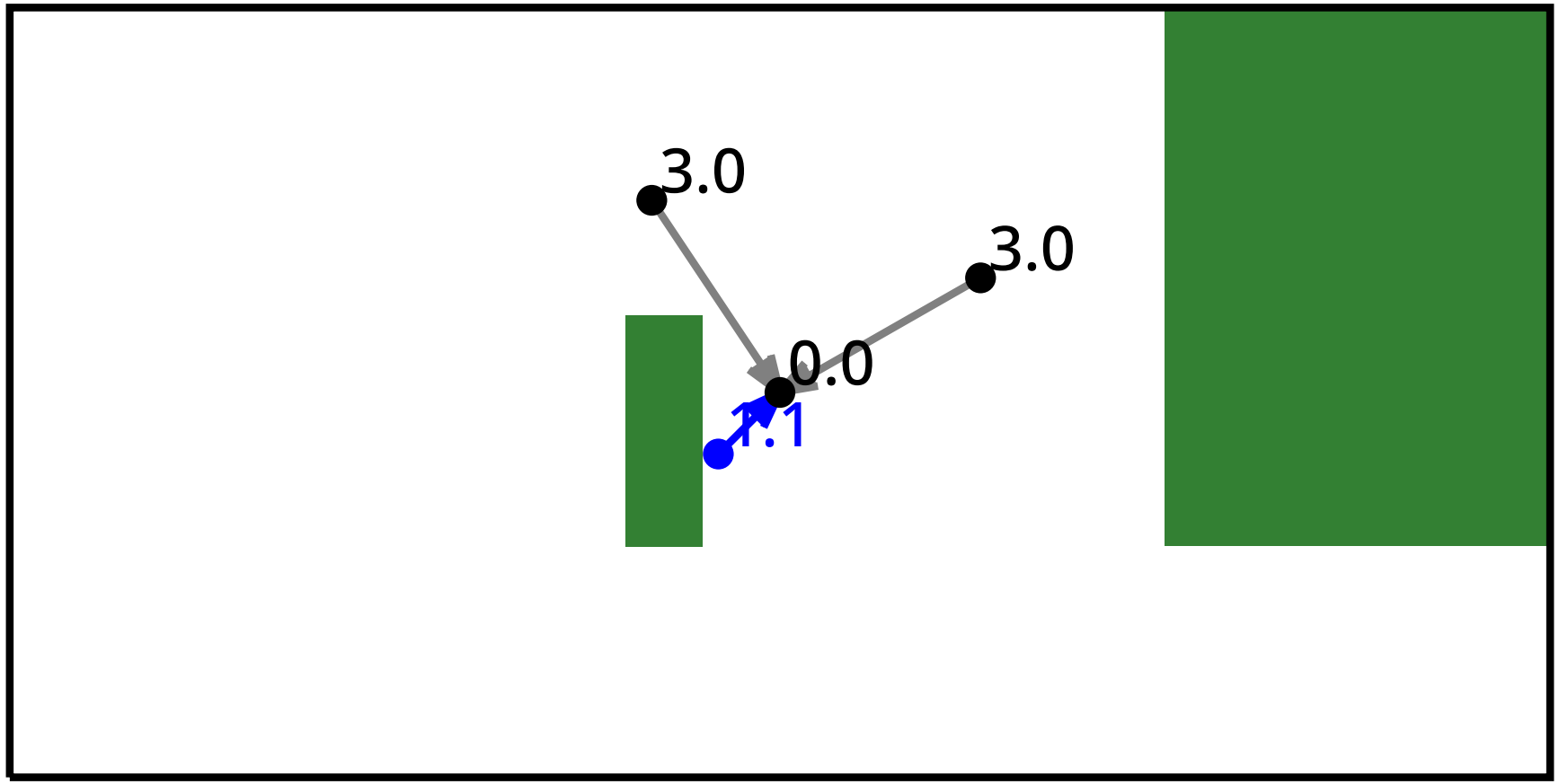
RRT*: Example 2

Find candidate parents for the new node; 2 candidates this time.



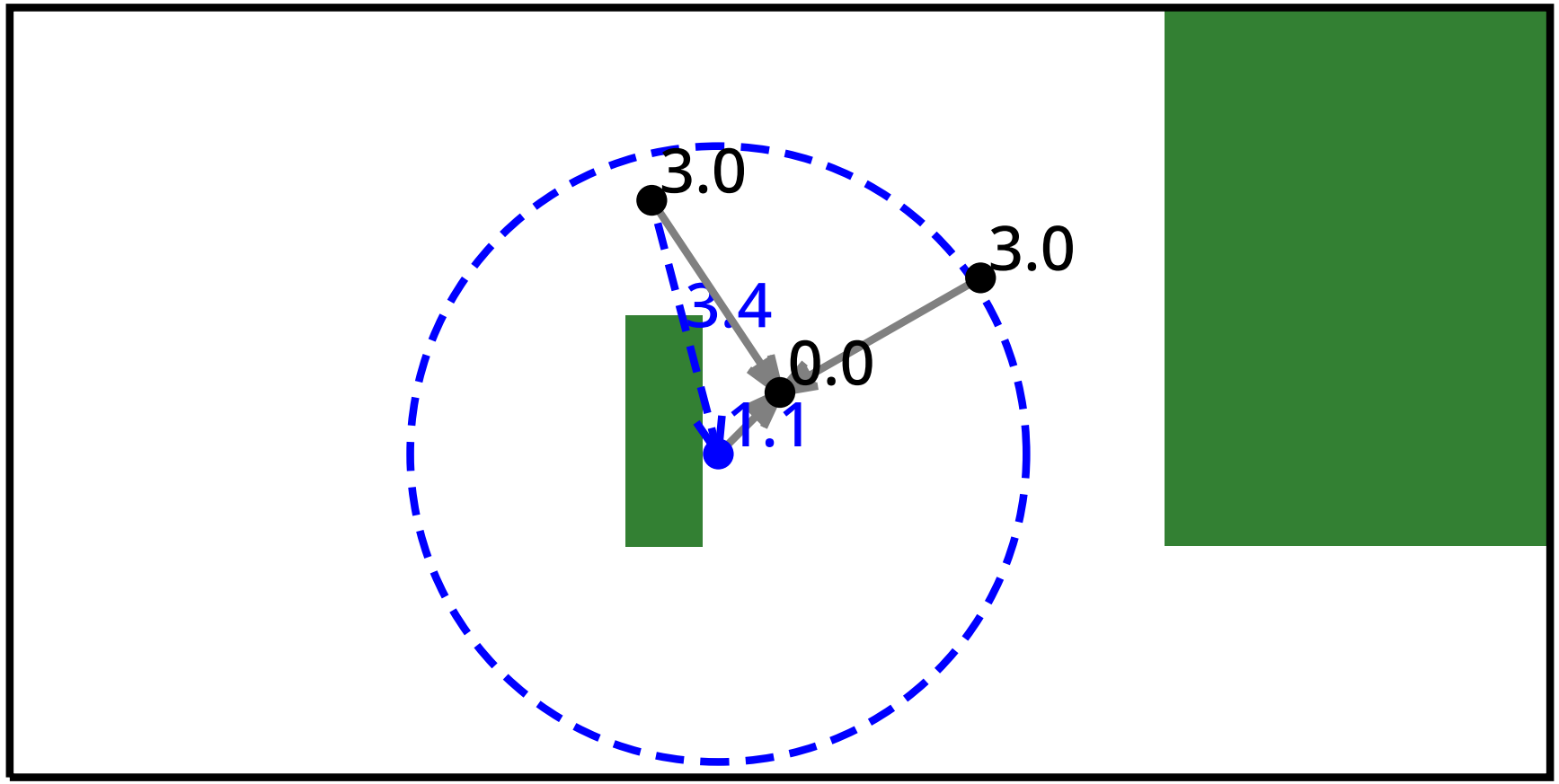
RRT*: Example 2

Choose the best parent for the new node.



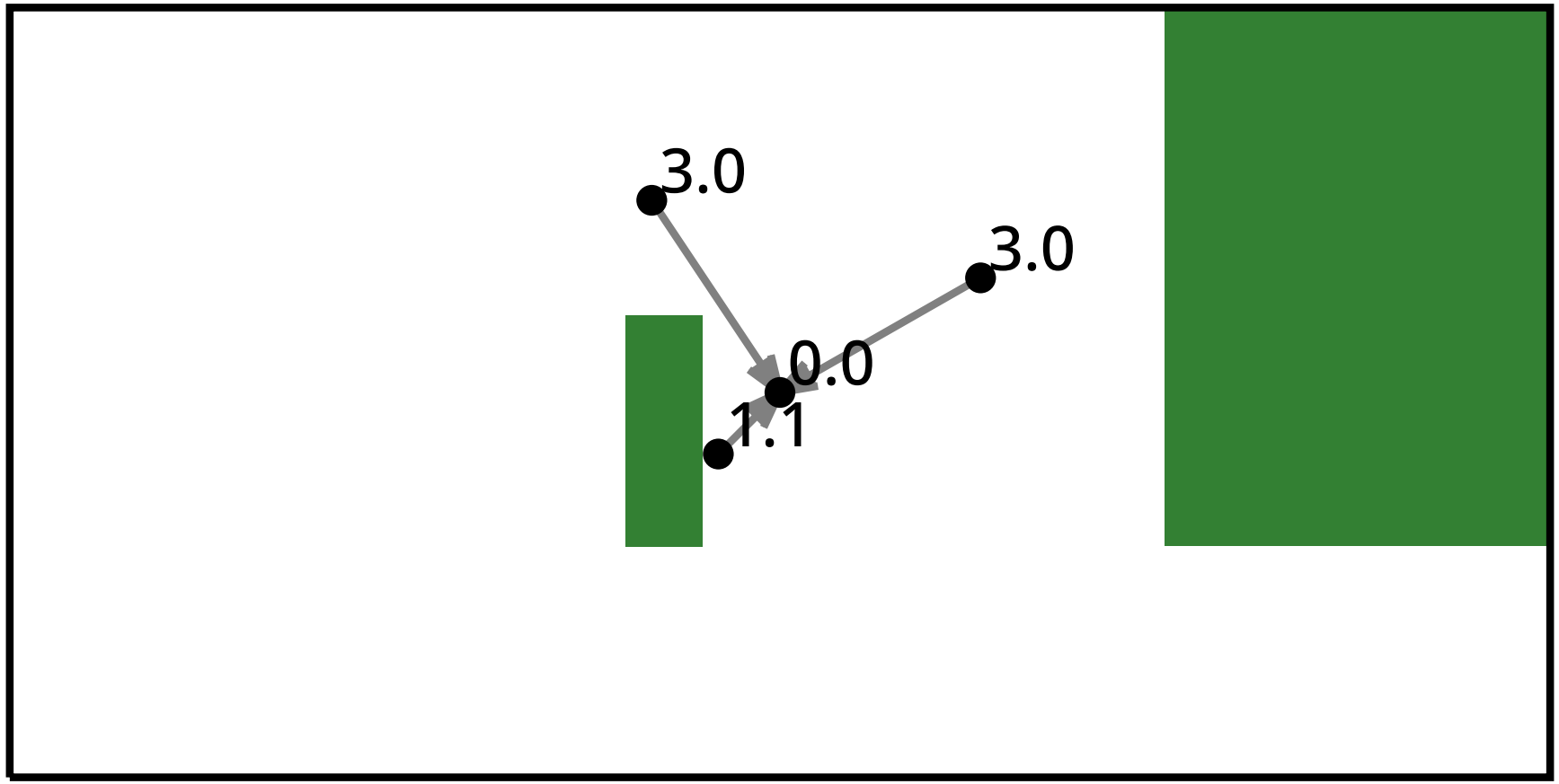
RRT*: Example 2

Find rewiring candidates near the new node; 1 candidate this time.



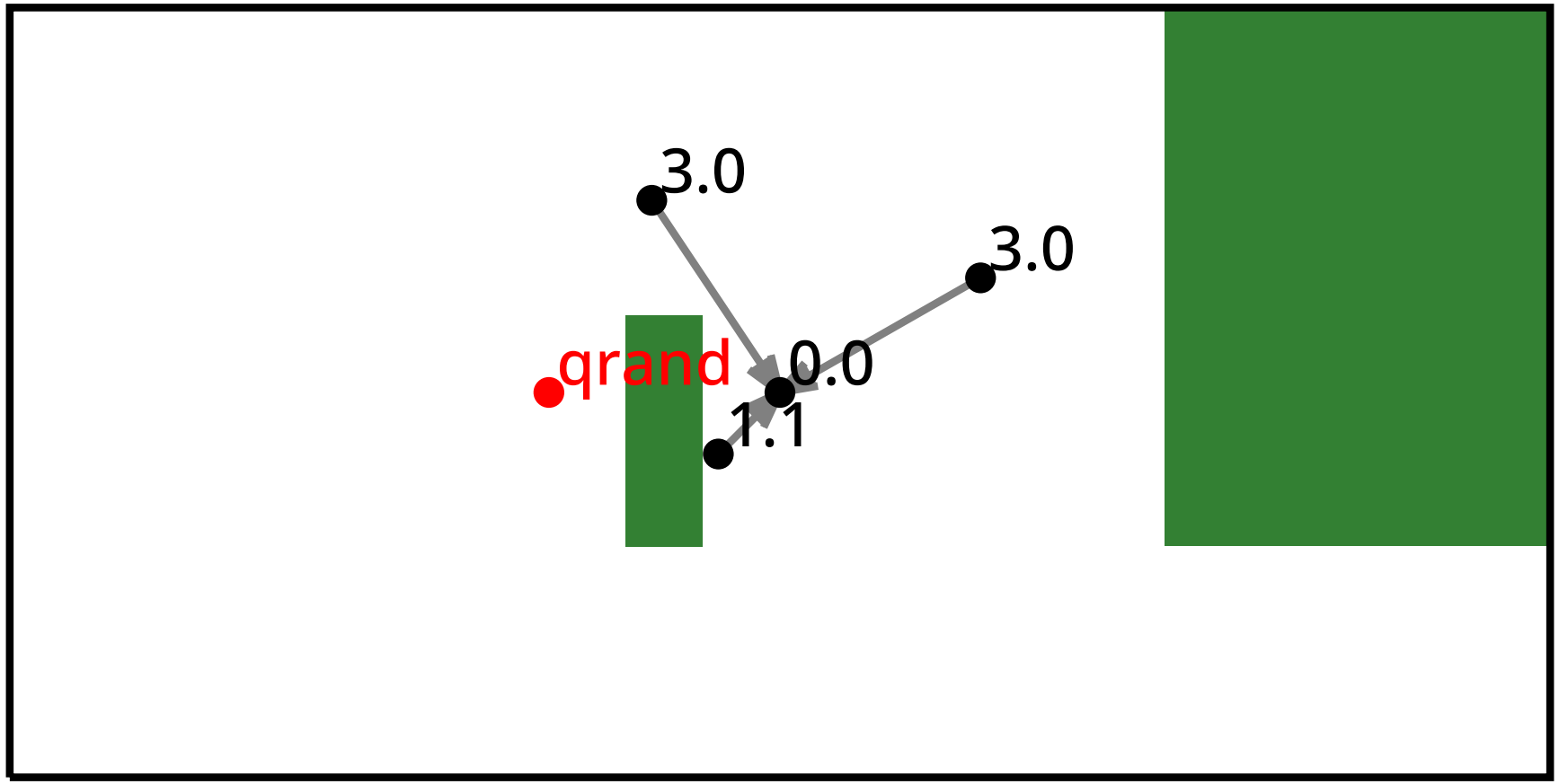
RRT*: Example 2

Rewire if needed; 0 changes this time.



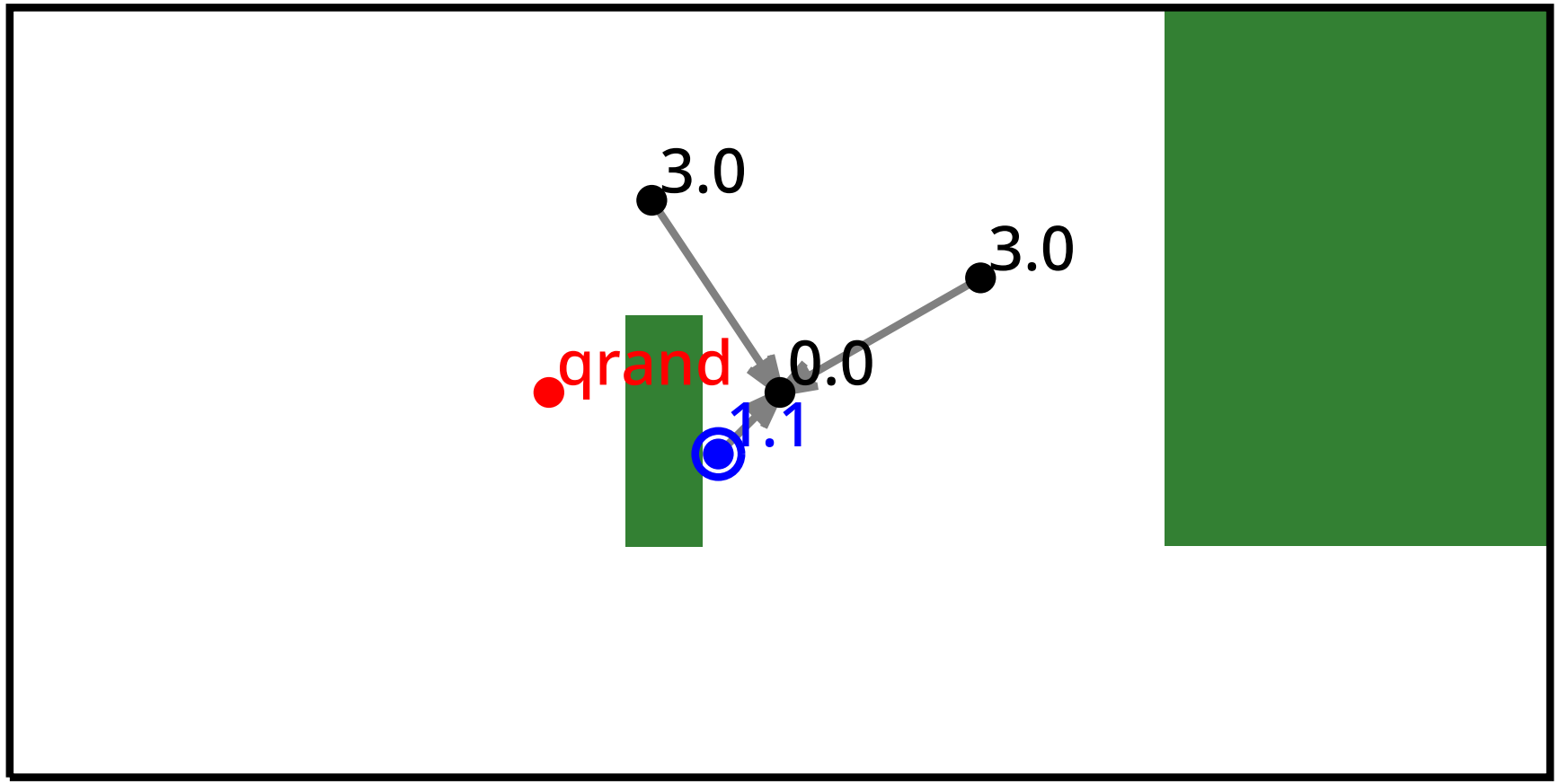
RRT*: Example 2

Choose a random sample.



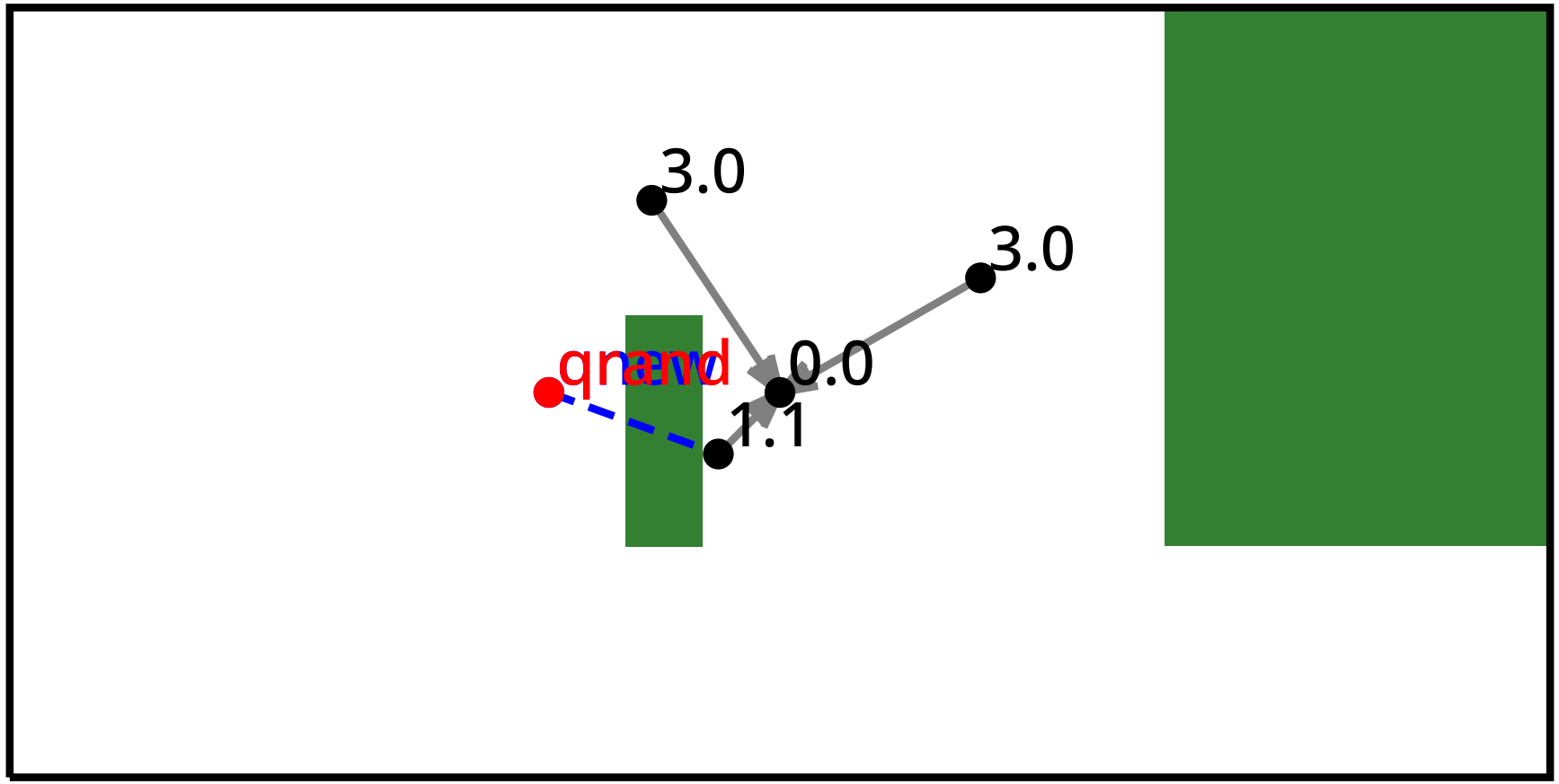
RRT*: Example 2

Find the nearest neighbor.



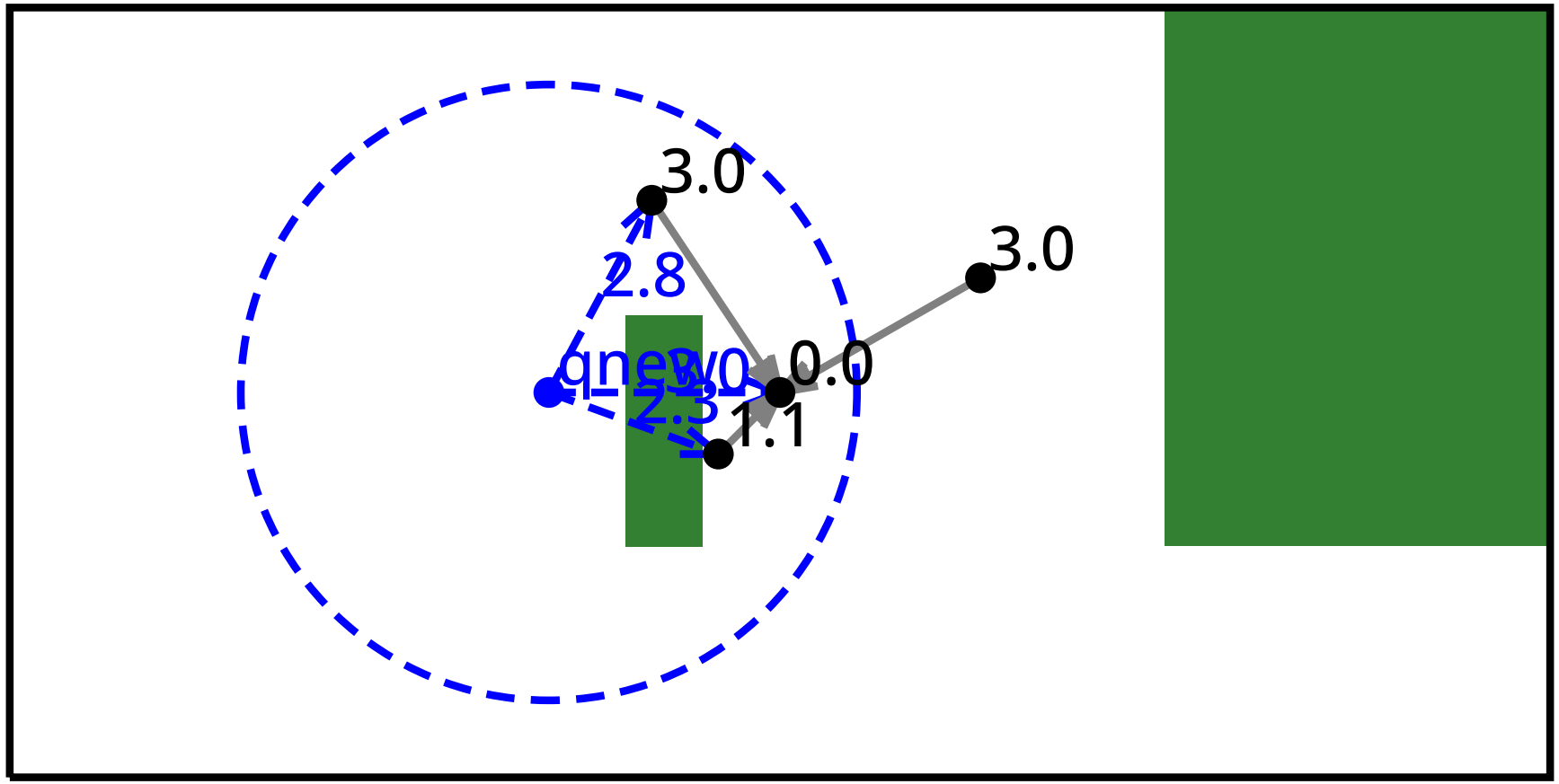
RRT*: Example 2

Extend from the nearest neighbor toward the sample.



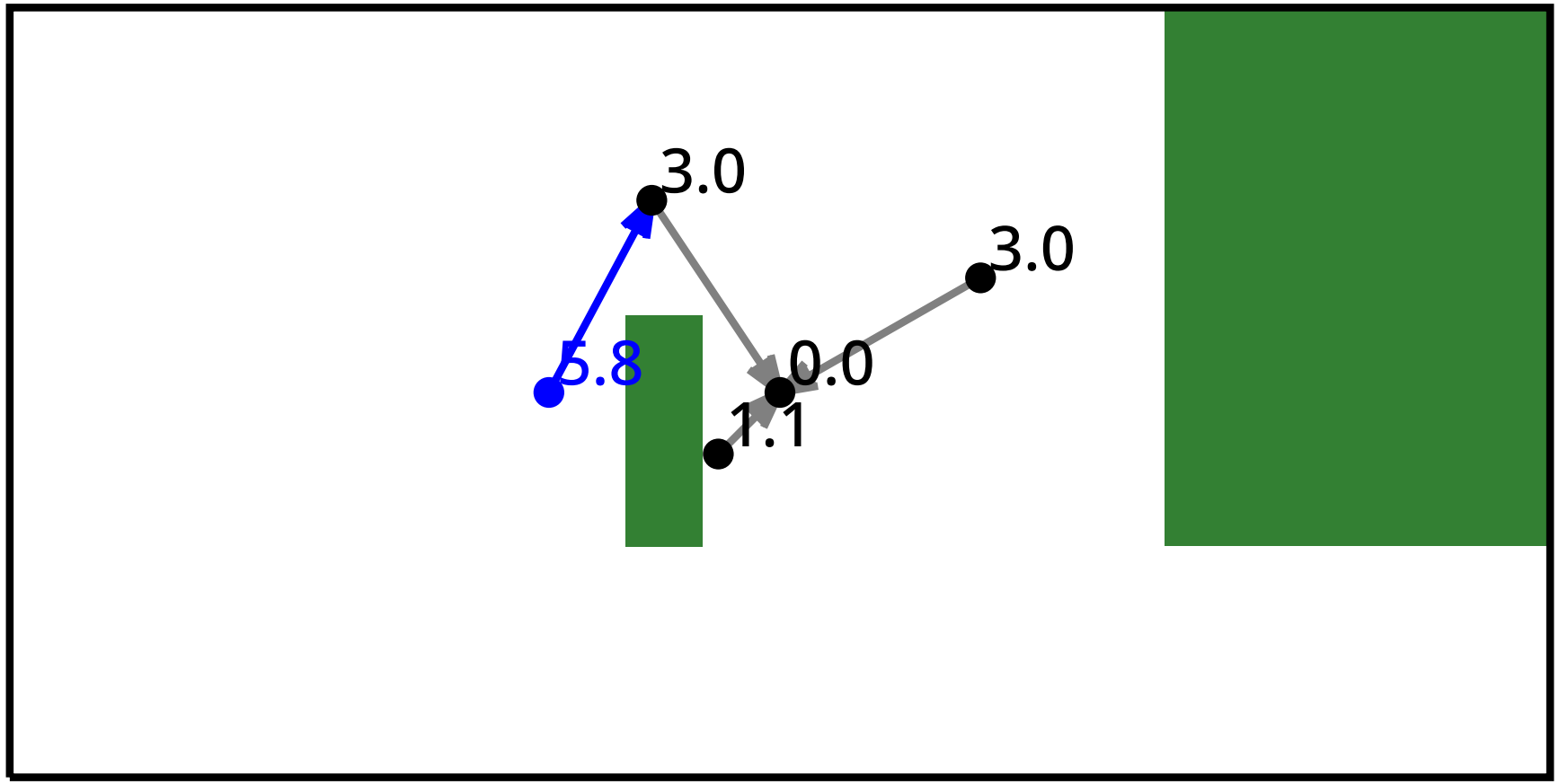
RRT*: Example 2

Find candidate parents for the new node; 3 candidates this time.



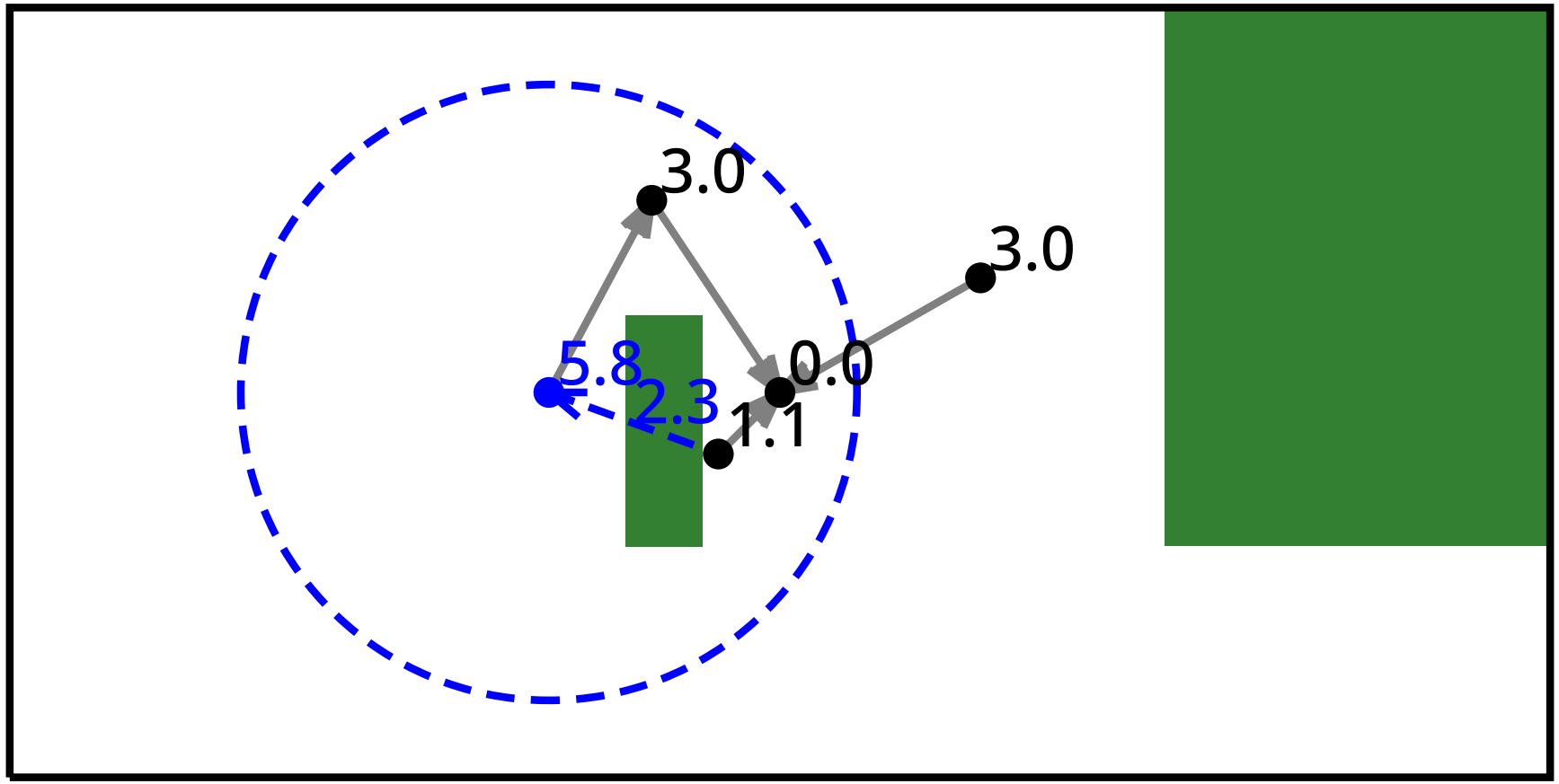
RRT*: Example 2

Choose the best parent for the new node.



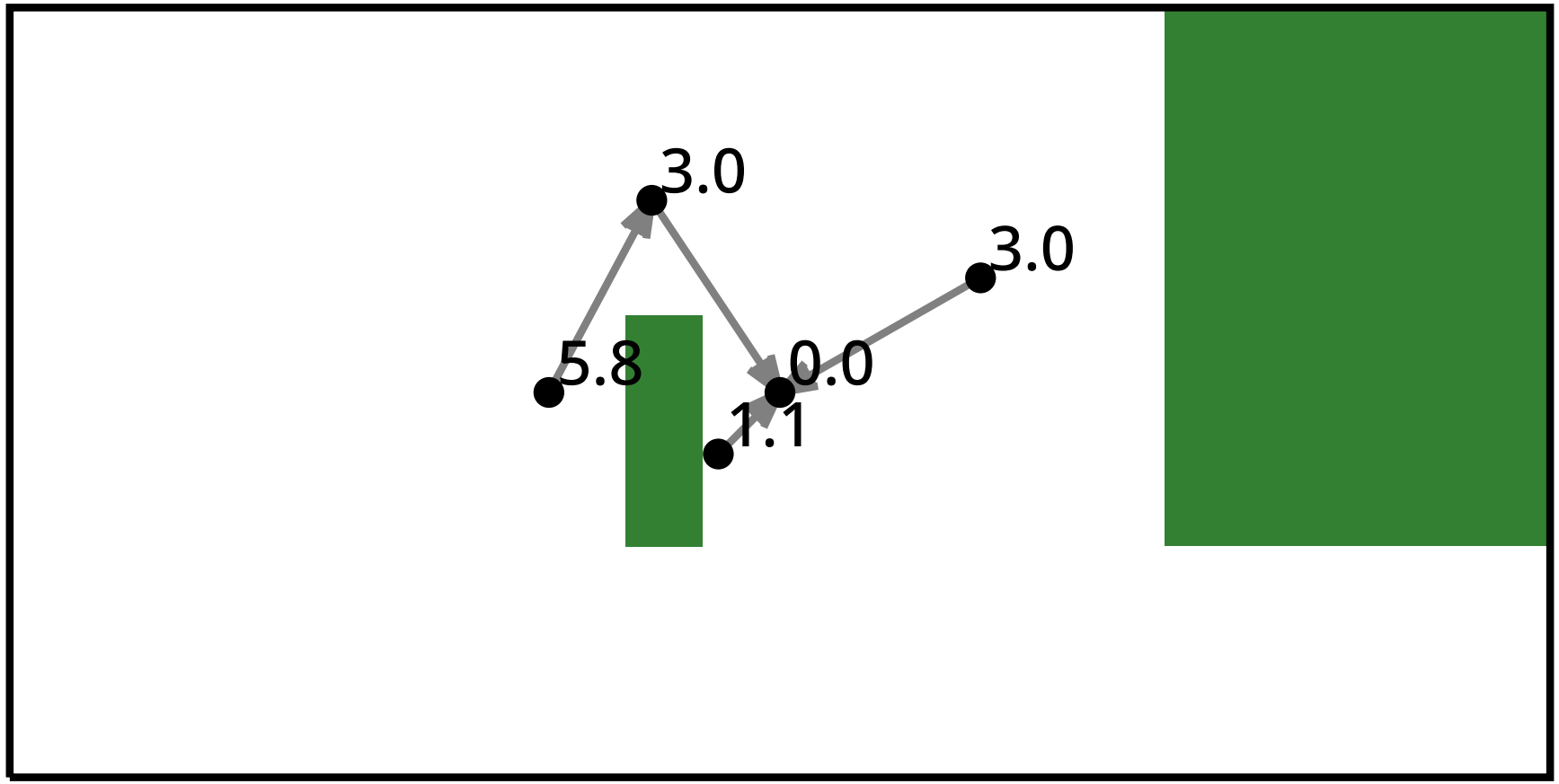
RRT*: Example 2

Find rewiring candidates near the new node; 1 candidate this time.



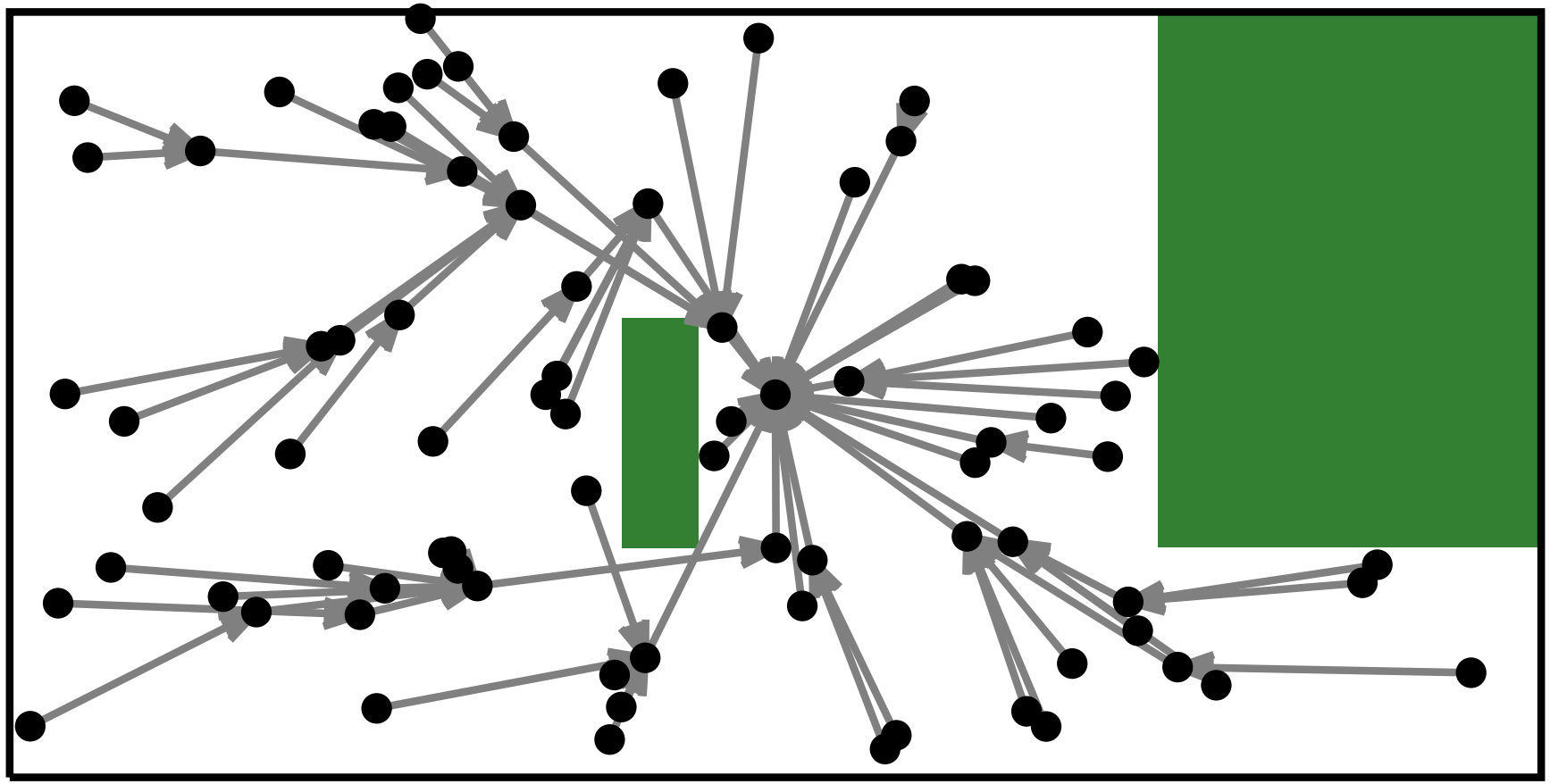
RRT*: Example 2

Rewire if needed; 0 changes this time.



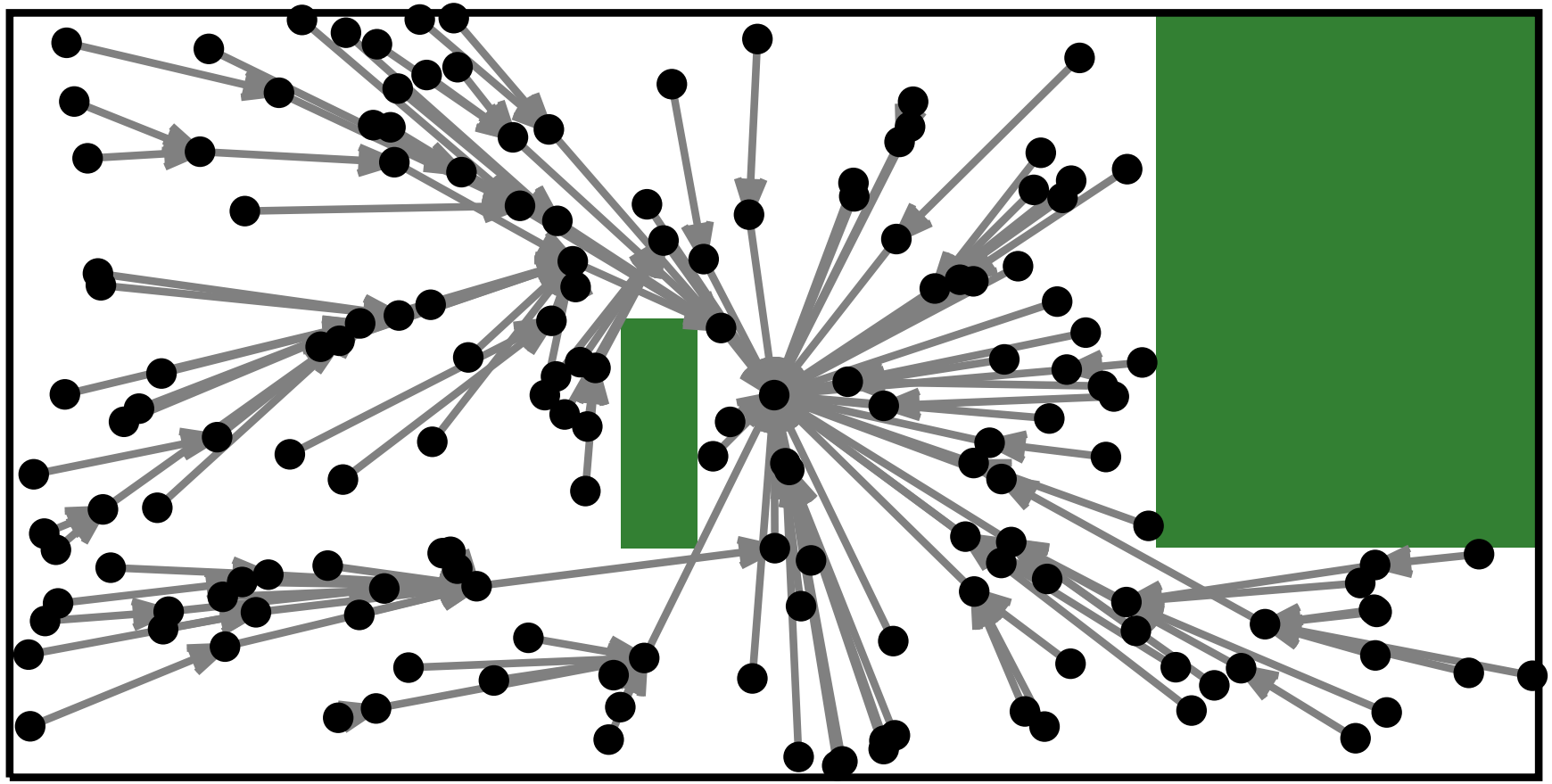
RRT*: Example 2

After 100 iterations



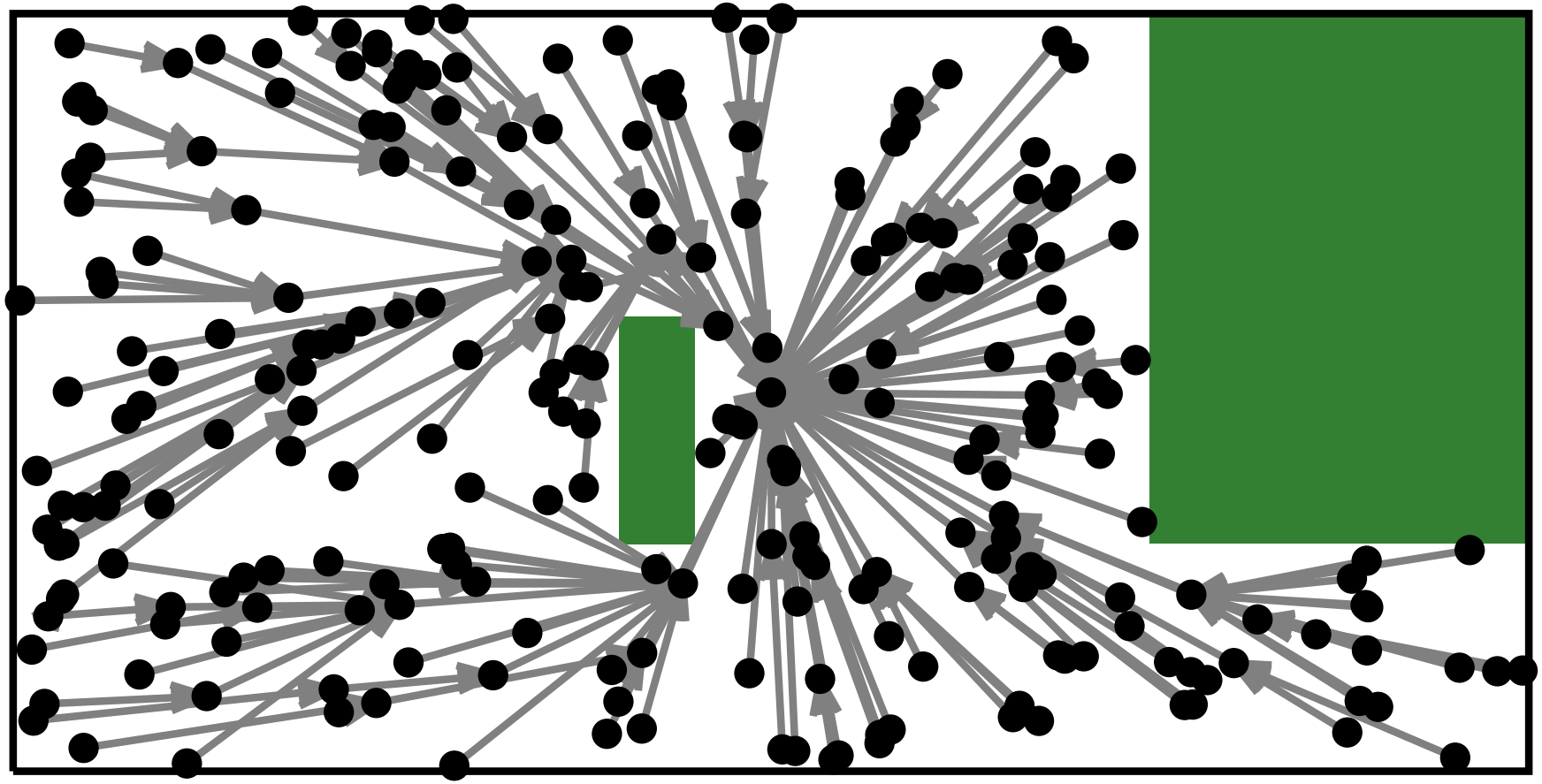
RRT*: Example 2

After 200 iterations



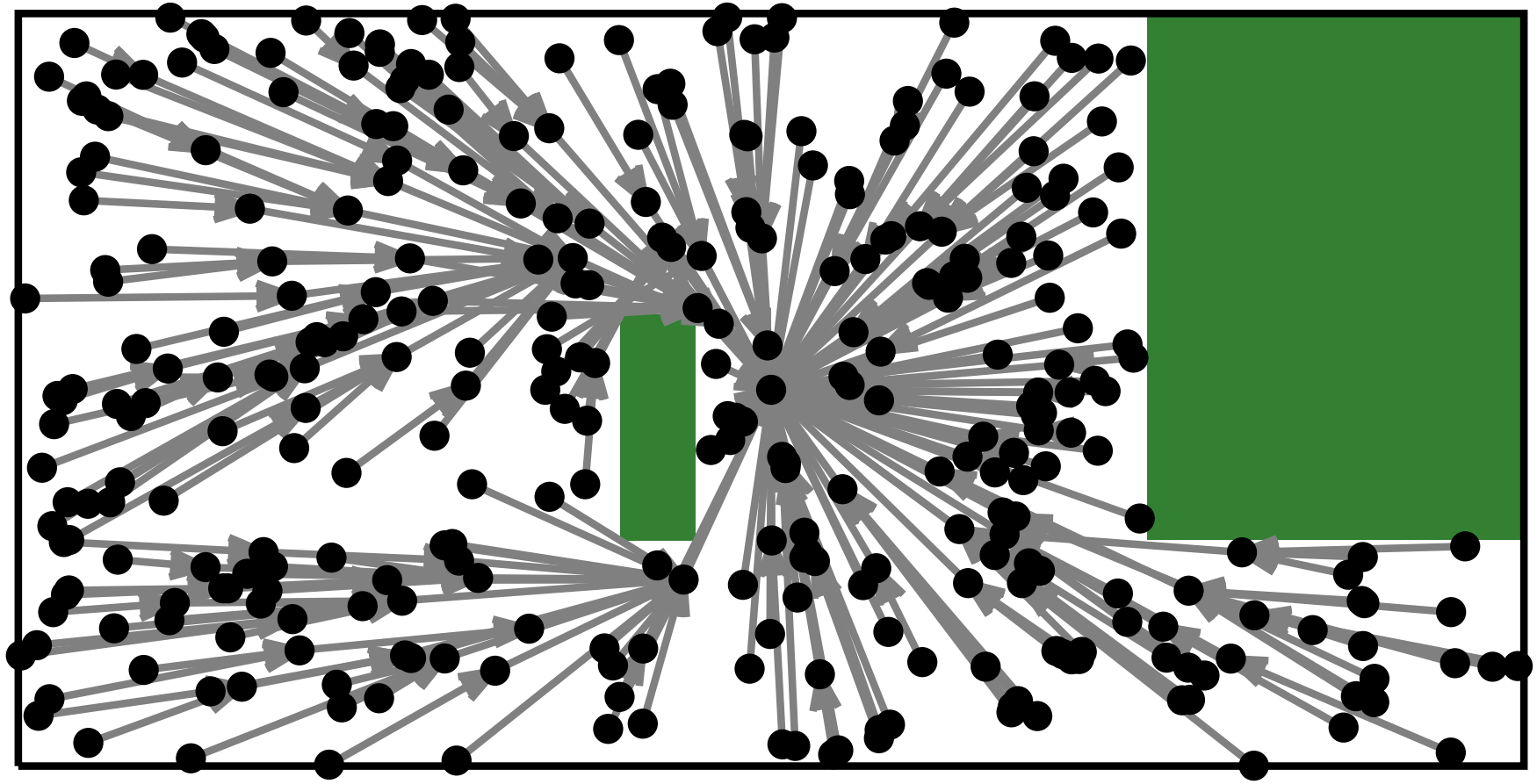
RRT*: Example 2

After 300 iterations



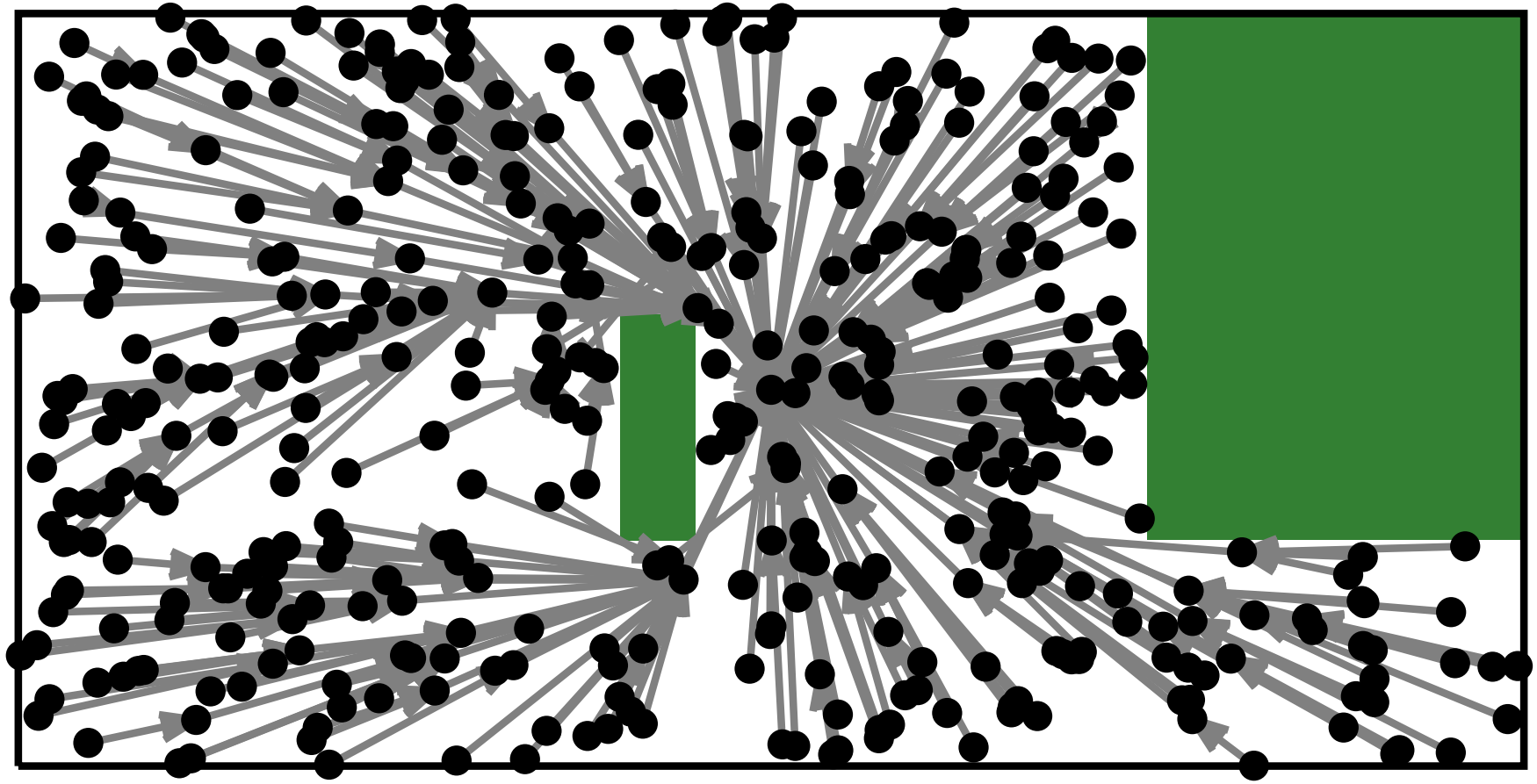
RRT*: Example 2

After 400 iterations



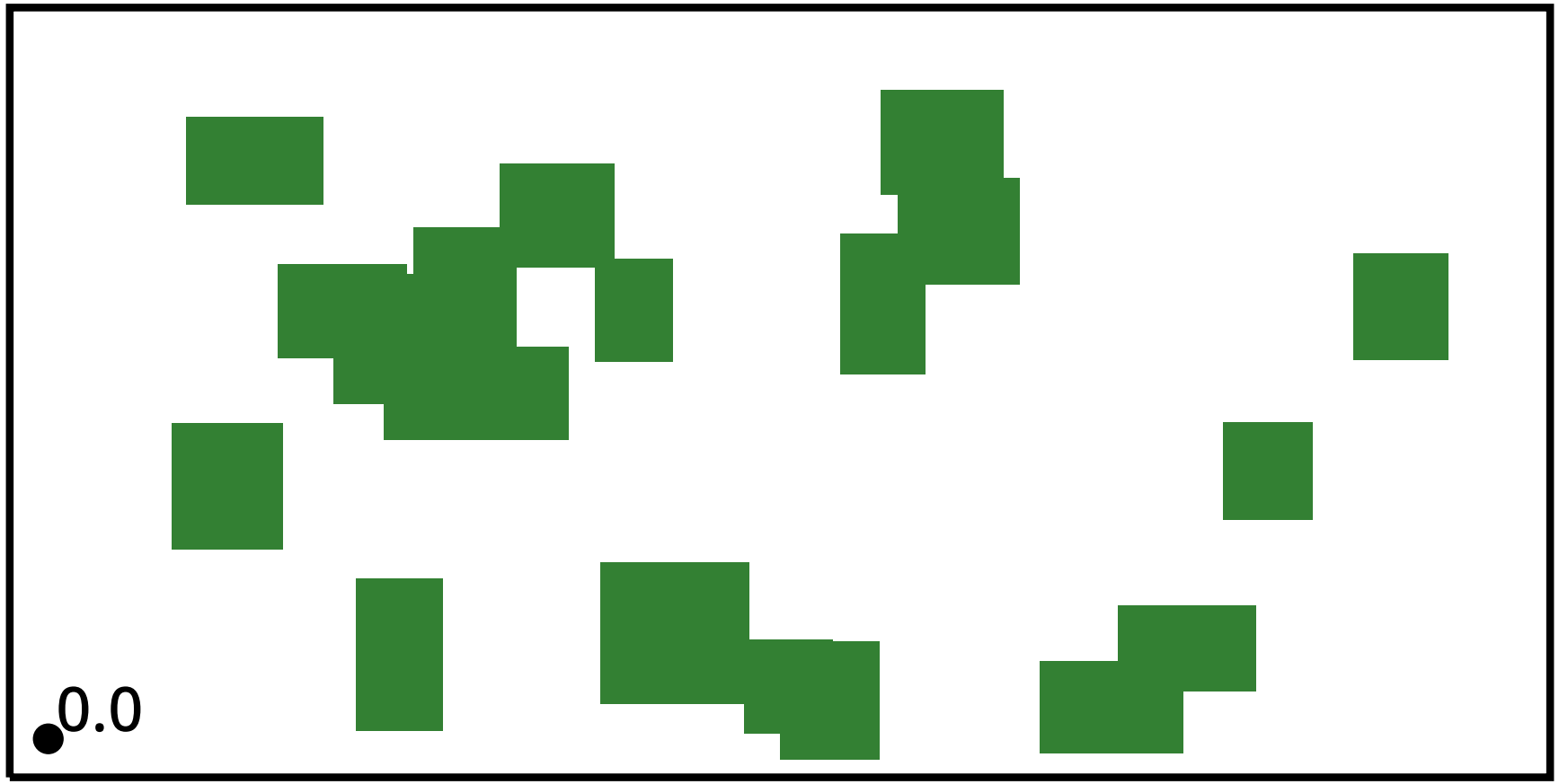
RRT*: Example 2

After 500 iterations



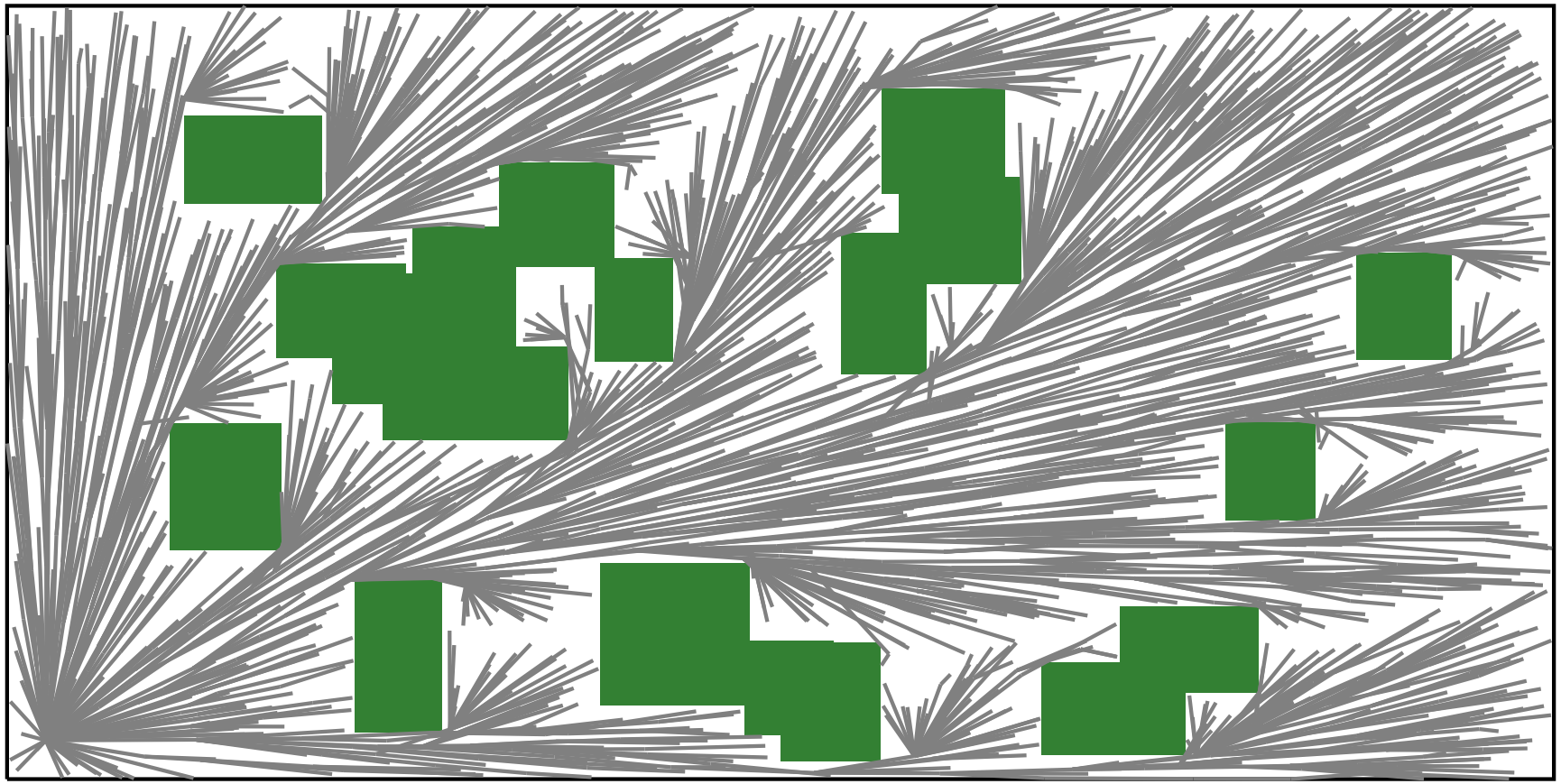
RRT*: Example 3

Start from a tree with just the start configuration.



RRT*: Example 3

After 3000 iterations



Asymptotic optimality

Definition A probabilistically complete motion planner is **asymptotically optimal** if the expected length of the solution it produces converges to the optimal length as the number of samples increases.

$$\lim_{n \to \infty} E[c(P)] = c^*$$

Theorem RRT* is asymptotically optimal.