Motion Planning

Configuration Spaces

Introduction

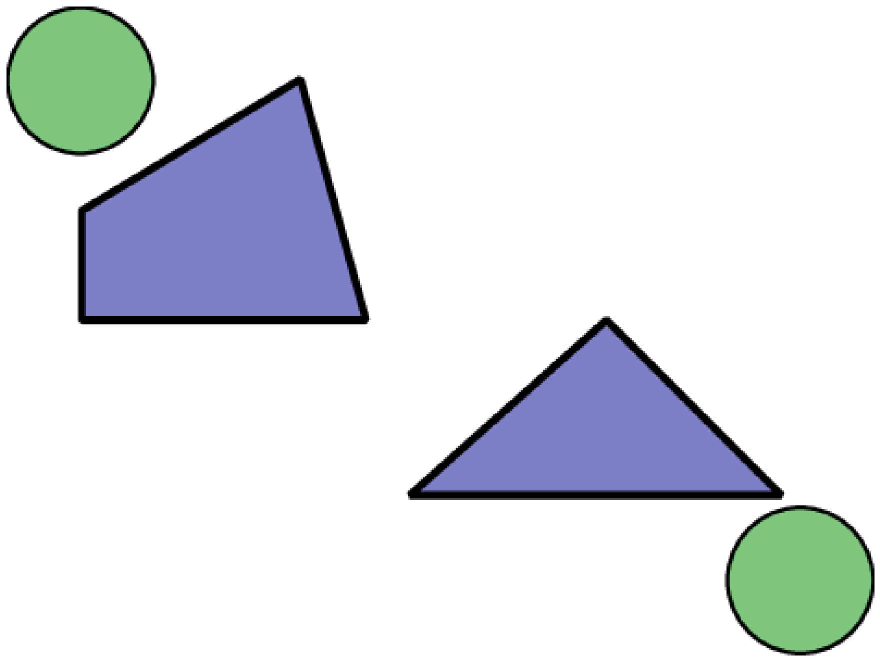
Motion planning algorithms generate continuous paths through the robot's **configuration space**.

Overview:

- The configuration space allows us to treat the robot as a point.
- For a typical motion planning problem, it is impractical to attempt to represent the complete obstacle boundary explicitly.
- Instead, use algorithms based on **sampling**. Two well-known sampling based algorithms are PRM and RRT.

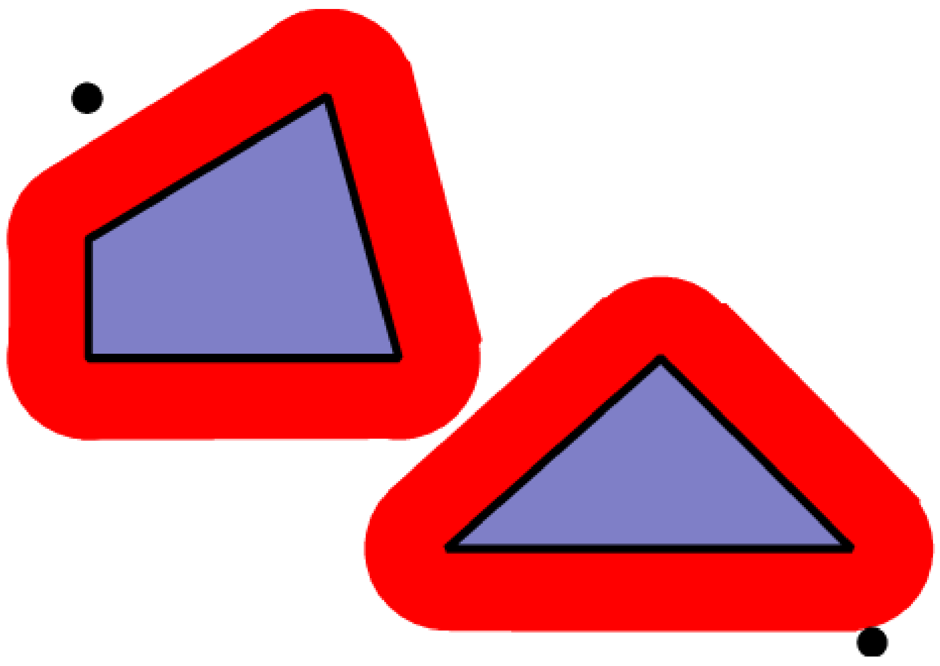
Motivation

Suppose we want to plan the motions for a circular robot amongst some known obstacles.



Intuition

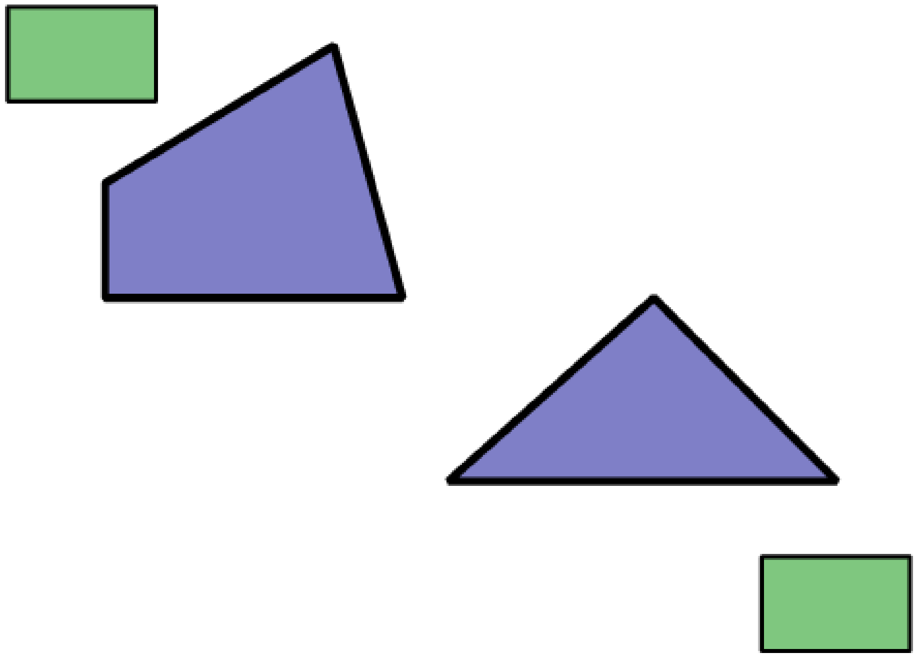
Informally, we want to shrink the robot down to a point and expand the obstacles by the same amount.



If we keep the center point out of the “expanded” obstacles, then the entire robot will stay out of the real workspace obstacles.

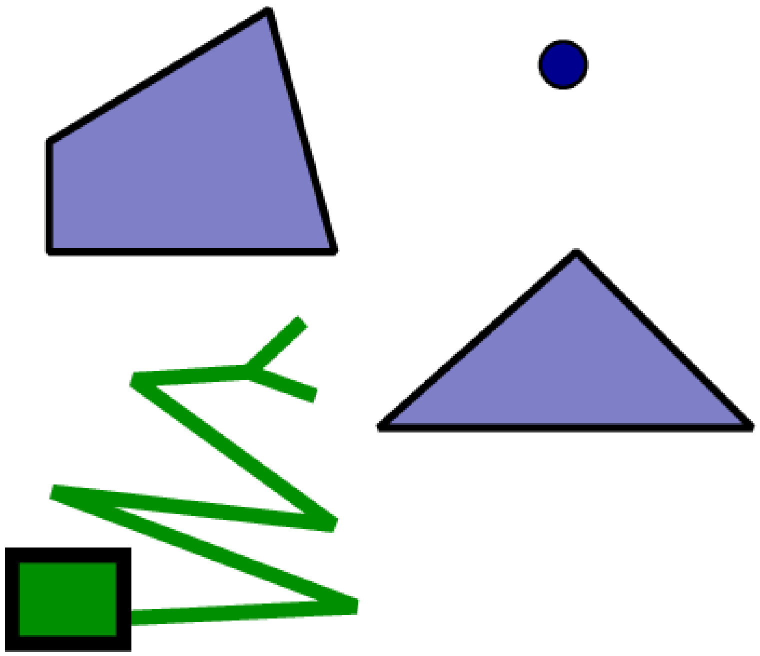
...but this intuition can only take us so far.

What if the robot is not a circle?



...but this intuition can only take us so far.

What if we want to plan motions for a multi-link manipulator to grasp a distant object?



Definition

The **configuration space** (C-space) $\mathcal{C}$ of a system contains one point for each combination of values for the robot's position, orientation, and internal joint positions.

Comments

The configuration space is a special kind of state space.

The configuration space should be a **topological space**. Informally, this means that it is meaningful to talk about the "neighborhood" of a configuration.

Many algorithms require the configuration space to be a **metric space**, which means that there is some reasonable definition of **distance** between pairs of configurations.

C-spaces for our examples

To find the configuration space for a given system, think about what parameters are needed to describe the robot's situation.

- Circular robot: We need just the x and y coordinates, so

$$\mathcal{C} = \mathbb{R} \times \mathbb{R}.$$

- Rectangular robot: Orientation is important, so

$$\mathcal{C} = \mathbb{R} \times \mathbb{R} \times S^1 = SE(2).$$

- Robotic arm: Five revolute joints, so

$$\mathcal{C} = S^1 \times S^1 \times S^1 \times S^1 \times S^1.$$

Common C-spaces

There are some C-spaces that occur in many different contexts:

- $SO(2)$ ("special orthogonal group"): A robot that can rotate (but not translate) in the plane.
- $SE(2)$ ("special Euclidean group"): A robot that can translate and rotate in the plane.
- $SO(3)$: A robot that can rotate in 3-space.
- $SE(3)$: A robot that can translate and rotate in 3-space.

(How many dimensions do each of these C-spaces have?)

Free space and obstacle space

The configuration space itself just describes where the robot is. It doesn't take the obstacles into account.

Obstacles:

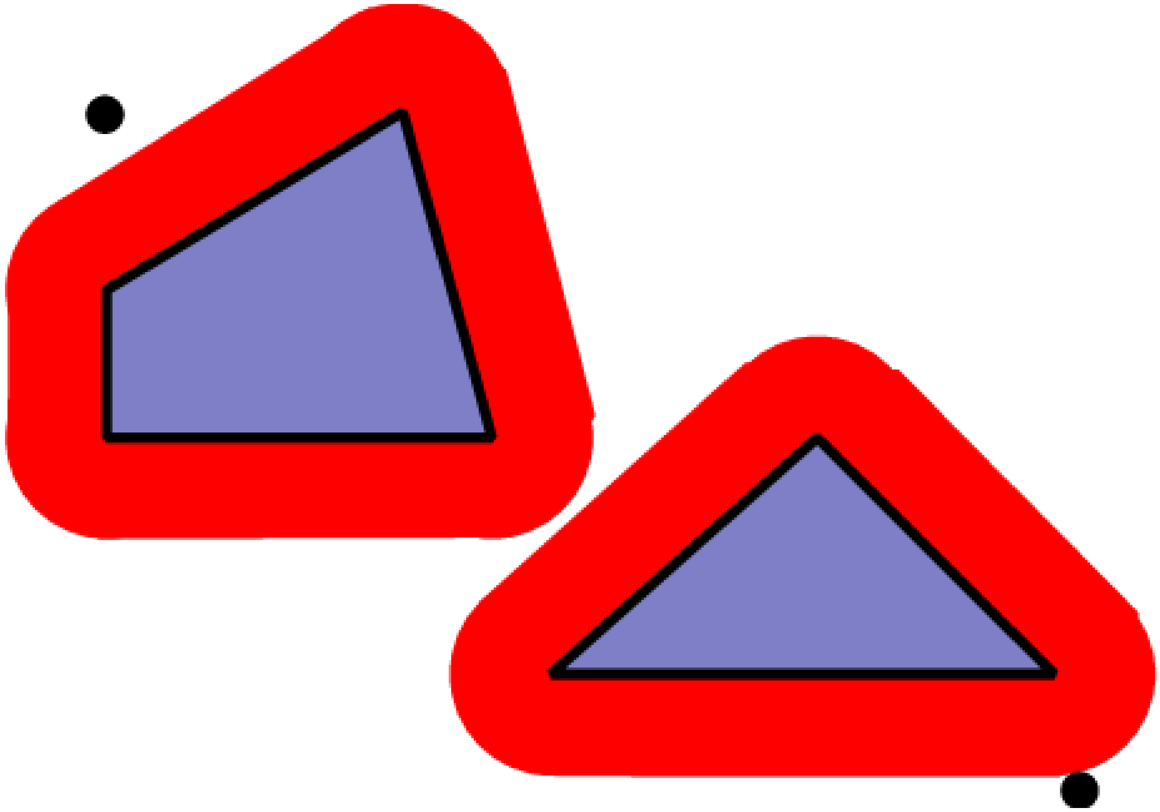
- An **obstacle configuration** is a configuration in which the robot is in collision with something in the environment, including possibly itself.
- The set of obstacle configurations is denoted $\mathcal{C}_{\text{obst}}$.

Free space:

- Everything else is a **free configuration**.
- The set of free configurations is $\mathcal{C}_{\text{free}} = \mathcal{C} - \mathcal{C}_{\text{obst}}$.

Back to our examples

For the disc robot:



Back to our examples

For the rectangle robot: