Localization 2: The Kalman Filter

Linear-Gaussian systems

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One special class of robot models that is very well-understood are the **Linear-Gaussian (LG)** systems.

1. **Linear transitions**: $x_{k+1} = Ax_k + Bu_k + G\theta_k$
2. **Linear sensing**: $y_k = Cx_k + H\psi_k$
3. **Gaussian noise**: Both θ_k and ψ_k are chosen **randomly** according to independent, zero-mean Gaussian densities with covariance matrices Σ_θ and Σ_ψ respectively.

A, B, C, G, and H are matrices with the appropriate dimensions.

Who cares?

LG systems are special because we can prove that the probability density

$$p(x_k \mid u_1, \ldots, u_k, y_1, \ldots, y_k)$$

is always a Gaussian, no matter what actions the robot takes nor what observations it receives.

This distribution can be represented by:

- μ_k: A state, representing the most likely location of the robot.
- Σ_k: A square covariance matrix, representing the “spread” of the distribution in each direction.