Localization 2: The Kalman Filter

Probabilistic localization

Introduction

Localization is the problem of determining and tracking the robot's position, relative to a map of its environment.

The **Kalman Filter** is a localization algorithm that maintains an estimate of the robot's state, expressed as a mean and covariance matrix.

The Kalman Filter is:

- passive
- local
- probabilistic

For linear-Gaussian systems, it maintains an **exact** representation of the probability of the current state.

Actions subject to error

Recall how we can model actions that experience error.

State transition function:

$$x_{k+1} = f(x_k, u_k, \theta_k)$$

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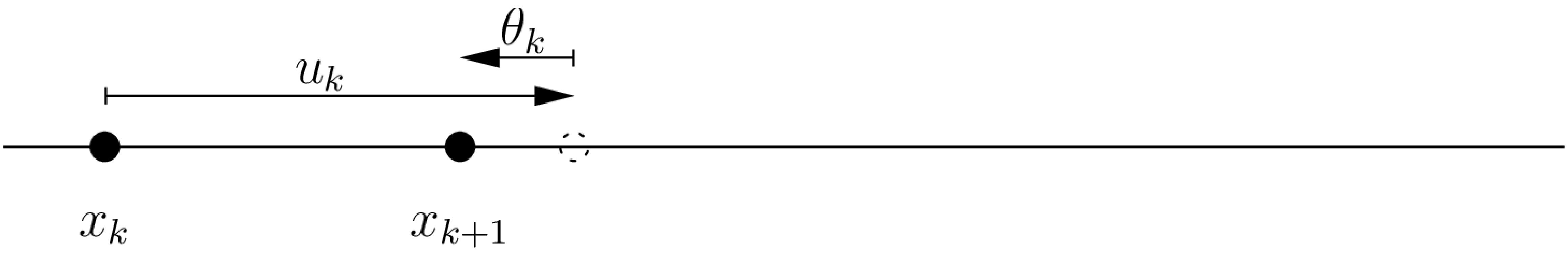
	element	**space of all**	**intuition**	**known?**
State	x	X	Where is the robot?	no
Action	u	U	What did the robot do?	yes
Action Error	θ	Θ	How accurately?	no

Simple example

A one dimensional system with additive actions and additive error:

$$X = U = \Theta = \mathbb{R}$$

$$x_{k+1} = x_k + u_k + \theta_k$$



Observations

Remember how we can model **observations** that provide partial information about the current state, subject to error.

Observation function:

$$y_k = h(x_k, \psi_k)$$

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	element	space of all	intuition	known?
Observation	y	Y	sensor data	yes
Observation Error	ψ	Ψ	How accurately?	no

Example continued

Example: Same as previous, with:

$$Y = \psi = \mathbb{R}$$

$$y_k = x_k + \psi_k$$

Probabilistic Localization

Problem: The robot does not generally know x!

- Known: history of actions and observations in the past
- Not enough detail in the history to compute x exactly

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Solution: Compute a **probability density over the state space**:

$$p(x_k \mid u_1, \ldots, u_k, y_1, \ldots, y_k)$$

This is **expected** or **average case** reasoning.