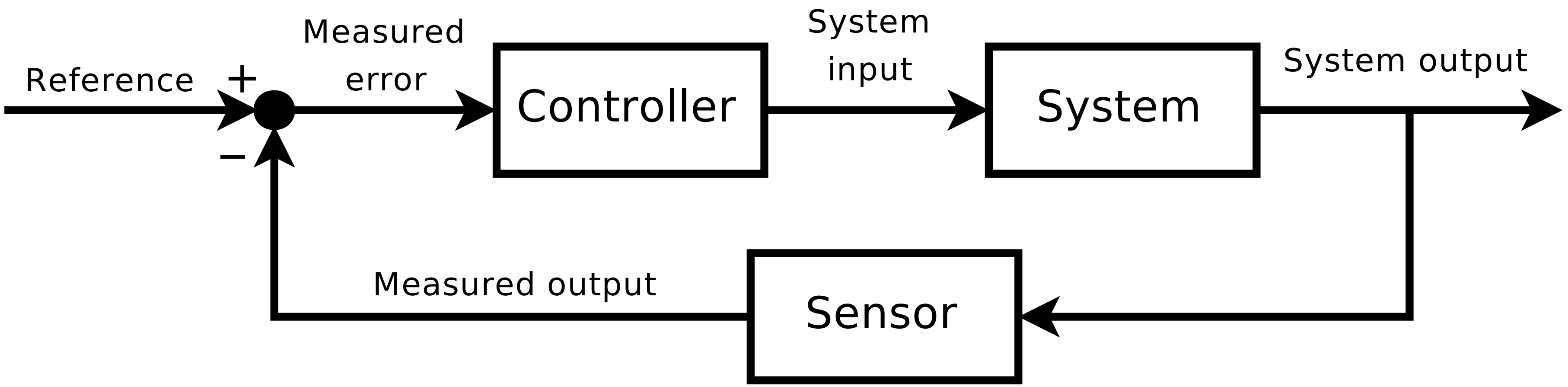
PID Control

Introduction to Control

Introduction

Definition

Control theory refers to methods for regulating the behavior of dynamical systems.



Example

A differential drive robot rotating in place:

- $X = [0, 2\pi)$ (all orientations)
- $x(t)$ (orientation at time t)
- $U = \mathbb{R}$ (all possible angular velocities)
- $u(t)$ (angular velocity at time t)
- $\dot{x} = u$

Goal

The goal is to **stabilize** the state to a **set point**.

The **error** is the difference between the state and the set point:

$$e(t) = s - x(t)$$

We want to choose actions that drive the state to the set point and keep it there.

Is this easy?

In the rotation example, suppose:

- Starting state: $x(0) = \pi/2$
- Set point: $s = 0$
- System model: $\dot{x} = u$

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Choose an **open loop control law**:

$$u(t) = \begin{cases} -\pi/4 & \text{if } \; t < 2 \\ 0 & \text{if } \; t \geq 2 \end{cases}$$

Result:

After $t = 2$, the system stays at the set point $x(t) = 0$.

Does this work?

Maybe, if the system is perfectly modeled and very predictable.

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But usually, we don't fully trust the system to obey the system model, due to **unmodeled disturbances**.

In our example:

- acceleration
- wheel slip
- miscalibration of the motors
- mismeasurement of the wheels
- uneven floor
- small delays executing the command
- ...

Feedback

Feedback controllers (state-feedback policies) can respond to states that we didn't expect to reach.

Basic idea: Actions depend on states:

$$\pi : X \to U$$

Back to our example

In the rotation example, suppose:

- Starting state: $x(0) = \pi/2$
- Set point: $s = 0$
- System model: $\dot{x} = u$

Choose an **closed loop control law**:

$$u(x) = -x$$

Result:

Result: From **any** state, the system moves toward the set point.