Multi-ASV Trajectory Planning for Enhanced Water Quality Sampling

Ibrahim Salman[1], Nare Karapetyan[2], Jason M. O'Kane[3], Ioannis Rekleitis[1]

***Abstract*— In this paper we present an integrated system for observation of transient freshwater phenomena, mainly Harmful Cyanobacteria Blooms, using a fleet of Autonomous Surface Vehicles (ASVs) fitted with onboard in-situ water quality sensors. Automated water quality sampling is often done following a predetermined trajectory, aiming to achieve the most cost effective representative coverage. We build on our previous work — the skeleton-of-skeleton technique, which selects sampling points representative of the body of water. Given a shared depot, the goal of the proposed algorithm is to produce trajectories for each ASV such that each sampling point is visited only once, thereby minimizing the traversal time for each robot and optimizing the operational timeline of the entire fleet. We formulate this NP-hard problem within the framework of the Multiple Traveling Salesperson Problem (mTSP), and use heuristics to address it. Water quality data was collected through multiple field deployments. Our experiments highlight the scalability of the automated system and are foundational in developing water quality sampling strategies.**

I. Introduction

Environmental monitoring tasks, such as water sampling and bathymetric mapping, are pivotal in studying hydrological processes. They are also instrumental in identifying early indicators of harmful toxins like Harmful Cyanobacteria Blooms (HCBs). Traditionally, these tasks have been resource-intensive, time-consuming, and occasionally hazardous as they have been primarily carried out manually by humans. Therefore, automating such processes would enhance the efficiency and safety of monitoring operations [1], [2]. As an example of such automation in practice, Fig. 1 shows an AFRL Jetyak ASV [3], collecting water quality measurements as it traverses through a lake.

The prevalent sampling pattern used in systematic environmental data collection is the "lawnmower" pattern. This method involves a vehicle moving in parallel lines, akin to a lawnmower, to capture samples across an entire region. However, the unstructured nature of lakes combined with the resource constraints of ASVs call for a more practical approach to sample collection. Multi-ASV methods present a promising alternative [4]–[6]. Nevertheless, the typically slow-changing nature of water quality measurements [7] combined with a focus on near-shore (littoral) areas necessitates a more innovative coverage strategy.

In previous work, we introduced the skeleton-of-skeleton algorithm, which leverages the medial axis of the target region to guide an ASV's trajectory [8], [9]. This algorithm systematically covers the target water body by traversing points equidistant to the medial axis and the water boundaries. In the presence of islands, the skeleton-of-skeleton algorithm yields a set of disconnected paths, leading to a major route around the lake and additional paths around each island [9].

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[1]University of South Carolina, Columbia, SC, USA. ijsalman@email.sc.edu, yiannisr@cse.sc.edu.
[2]Woods Hole Oceanographic Institution (WHOI), Woods Hole, MA 02543, USA. nare@whoi.edu
[3]Texas A&M University, College Station, TX. jokane@tamu.edu

Fig. 1: AFRLJetyak ASV collecting water quality data along trajectory at Lake Wateree, SC, USA.

Building on these foundations, the present study designs a complete system aimed at enhancing water sampling efficiency through the utilization of multiple ASVs. First, we use the skeleton-of-skeleton algorithm, which generates disjoint linear contours representing the environment along the areas of interest. Next, we formulate the problem as an mTSP instance, and solve it using heuristics to find the optimal routes that visit all these sampling points in the minimum deployment time. Finally, we deploy the resulting paths on ASVs on the surface of large lakes, collecting water quality data to aid in understanding and detecting HCBs.

As such, the contributions of our system are as follows:

- Extending the skeleton-of-skeleton algorithm to a multi-agent system.
- Extensive field deployments on the surface of lakes in South Carolina collecting water quality data using a YSI's EXO2 Multiparameter sonde [10] and an OTT's Nitrate Sensor [11].

The field experiments demonstrate that our enhanced approach adeptly addresses the issues with disconnected paths and ensures thorough and optimized coverage of expansive water bodies. This approach holds promise for practical water sampling tasks, including potential HCB detection.

II. Related Work

Research on coverage path planning has been ongoing for a considerable time, with several studies delving into the subject [12], [13]. Traditional approaches usually involve utilizing a grid-based pattern, such as the seed-spreader algorithm [14], the boustrophedon algorithm [15], or the lawnmower pattern. These methods have been expanded to suit various environments, including known [16] or unknown [17] settings, and for single [18], [19] or multiple [4], [5] ASVs that operate under Dubins constraints. The fundamental goal of these techniques is to ensure that the vehicle covers every open space in the area. However, when there are resource constraints, such as a limited operation period relative to the environment's size, it becomes impractical to achieve complete coverage. For instance, an Unmanned Aerial Vehicle (UAV) might only have a 20-minute operational window, while many ASVs can operate for several hours. To overcome this limitation, a prevalent approach is to guide the ASV to areas of interest using information-driven coverage [20], [21]. This approach has proven helpful in mapping coral reefs [22] and locating the source of plumes [23].

A common theme through much of the related work is using solutions that rely on solvers for TSP or similar NP-hard problems. When dealing with a single robot coverage problem, some works [16] have defined the problem in a manner that allows it to be solved in polynomial time, given certain assumptions. Lopez et al. [24] also focus on monitoring water quality in large bodies of water. Their work utilizes an adaptation of TSP, seeking to maximize (rather than minimize) the distance traveled to obtain thorough coverage. Nevertheless, when dealing with multi-agent systems, the problem is known to be NP-Hard [25]. Multiple approximation methods have been proposed to solve the complete area coverage problem [4], [5], [26]. Nevertheless, these methods are not suitable when the the boundaries of the coverage regions are unstructured.

When tackling the issue with unstructured boundaries, some works has been done to design patterns specifically suitable for coverage of riverine environments [1], [27]. These methods rely on the inherent structure of the rivers and the domain knowledge on what type of strategies are used by scientists for bathymetric mapping. However, these methods aim to perform a complete coverage depending on the type of the surveying sensors being used. When the resources are limited and the region has unstructured boundaries in our earlier work we have proposed a method that uniformly samples a very large environments using the concept of points equidistant from obstacles [2], [8].

The latter is based on the paradigms of computational geometry. The concept of curves in a geometric space, which denote free space and can serve as simplified representations of the space's topology, is commonly referred to by a number of different names, such as Medial Axis, Skeleton, Generalized Voronoi Graph, and Center of Maximal Disks [28], [29]. Generalized Voronoi Graphs (GVG) [30] have been widely used for various purposes, including exploring indoor spaces, navigation without exact localization [31], and reducing localization errors [32]–[34].

A similar boundary coverage problem with multiple robots [35] was addressed by reducing it to the k-rural postman problem, but in a context where boundaries near each other can form 'viewing channels' that allow the robots to observe multiple boundaries from a single vantage point. In contrast, our work uses the skeleton-of-skeleton algorithm [8] to select representative paths to cover rather than requiring viewing from within a specific range. Our work, in contrast, is targeted toward water bodies that cannot adequately be covered in full by a single robot because of energy and time limitations. Hence we propose to extend the unstructured area coverage problem by employing multiple robots to enhance the monitoring efficiency.

III. Problem Statement

This section formulates the planning problem as an mTSP, aiming to balance route coverage optimization with workload equity across a fleet of robots. The core objective is twofold: to reduce total travel distance, $\sum_{i=1}^{m} |\tau_i|$, for enhanced operational efficiency, and to normalize the distribution of route lengths, thereby ensuring a minimized deployment timeline through equitable task allocation among robots, $\min \mathrm{Var}(|\tau_i|)$, where $\{\tau_1, \tau_2, \ldots, \tau_m\}$ represents the set of trajectories for each robot.

Input

- m robots, all sharing the same depot. If $m = 1$, the problem is reduced to a classic Traveling Salesperson Problem (TSP).
- Skeleton-of-skeleton generated paths, discretized to a sequence of waypoints P delineating the routes to be covered.

Assumptions

Several assumptions are considered:

- A static environment, no dynamic obstacles.
- Robots have uniform speed and sensor capabilities.
- Robots share a single depot (start and end point).

Output

- A set of paths $\{\tau_1, \tau_2, \ldots, \tau_m\}$, with each trajectory $\tau_i : [0, T] \to \mathbb{R}^2 \times [0, 2\pi)$ representing the movement of the i-th robot.

IV. Proposed Approach

The proposed approach presents a complete system solution for automating the water sampling task using m number ASVs. We define the problem on a known environment described by an occupancy grid $M : R^2 \to 0, 1$, which is extracted from a satellite image. This approach is applicable to any problem where a set of disjoint paths needs to be traversed by m number of robots. The particular application we are investigating is consolidating the output of the skeleton-of-skeleton coverage path [8] into a set of trajectories to be traversed by m robots.

Our approach consists of two main phases:

(1) We utilize the skeleton-of-skeleton coverage algorithm [8] to generate efficient water sampling paths representative

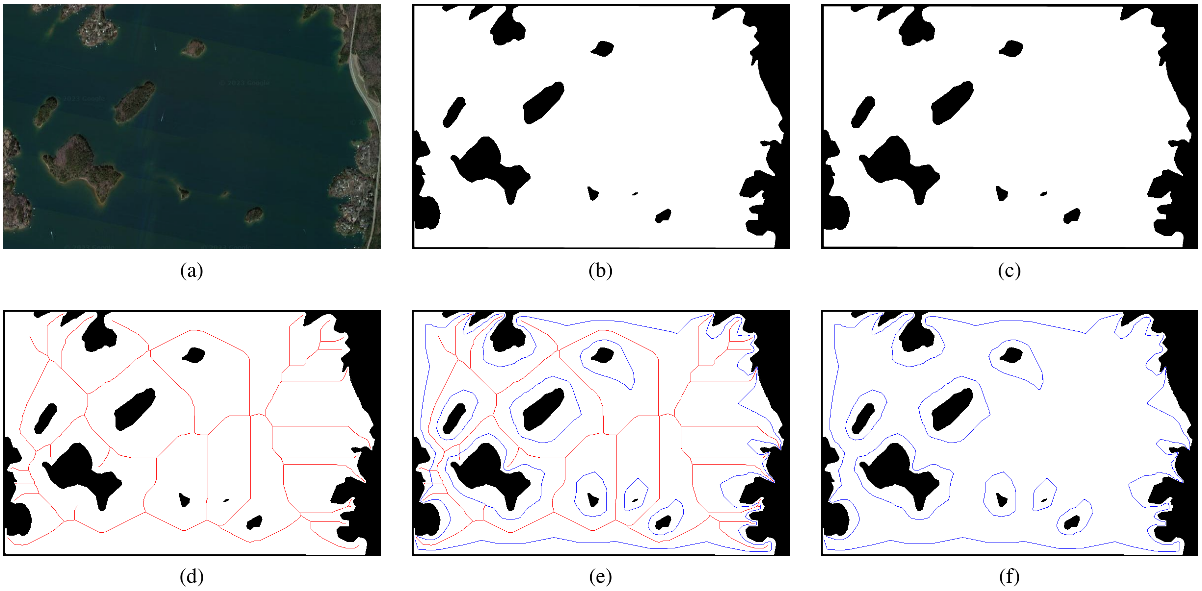


Fig. 2: Analysis of the Skeleton-of-the-skeleton algorithm at Lake Keowee, SC, USA. (a) Satellite view of the area. (b) Obstacle and free space binary map. (c) Enhanced safety by dilating obstacles in the free space map. (d) The main free-space skeleton is highlighted in red. (e) After trimming, both primary (red) and secondary (blue) skeletons. (f) Waypoints for ASVs generated from the skeletons.

of the selected area which the robots will follow given the layout of M. These patterns can be described as a sequence of n curves $C_1, ..., C_n$; where $C_i : R^2 \to [0, 1]$. We discretize each curve with step size t and approximate it as a sequence of points $C_i = \{p_{i1}, p_{i2}, ..., p_{ij}\}$.

(2) We formulate the NP-hard problem within the framework of mTSP to find m efficient paths that traverse all the discretized curves in the least time possible. The input is D, a distance matrix of obstacle-free shortest path distances of the sequences of points representing the curves C_i.

Algorithm 1

Require: D: the distance matrix, m: number of robots
if $m = 1$ **then**
  $T \leftarrow$ **TSP**(D)
  return T
end if
Search Params:
  Initial: Christofides
  Meta: GLS
$T \leftarrow$ **mTSP**$(D, m, Initial, Meta)$
return T

A. Skeleton of Skeleton

Here, we outline the Skeleton-of-skeleton approach [8].

1) Area of Interest: We first start by selecting a rectangular-shaped area of interest. We do so by noting the area's top left and bottom right GPS coordinates. We capture a satellite imagery snapshot, as shown in Fig. 2(a) aligning with the outlined method in [8].

2) Binary Map: For image processing purposes, it is easier to work with a landmass layer of the map, so we also get that which is later transformed into the black and white image (white for free space – water, and black for obstacles); see Fig. 2(b).

3) Safety parameters: In Fig. 2(c), the image is dilated and eroded using a safety distance (calculated based on the pixelArea we picked earlier) to ensure the ASV maintains a safe distance from docks and other near shore obstacles not present in the landmass layer. Further manual inspection and editing of the black and white map ensure that such objects are recorded.

4) Skeleton of free space: In Fig. 2(d), we capture the morphology of the free space by extracting the medial axis of the free space. We tested several variations of the skeleton algorithm. We found that the Lee skeleton algorithm [36] worked best in our case across different bodies of water and required less edge trimming. The skeleton is then trimmed eliminating edges that barely enter coves and removing small edges.

5) Skeleton of skeleton: The medial axis between the original Skeleton in Fig. 2(d) and the dilated environment in Fig. 2(c) is generated, resulting in one (if there are no islands) or more closed-curve ASV trajectories. Fig. 2(f) shows the final trajectory/trajectories in red. The generated trajectory visits points equidistant to the Skeleton and the obstacle boundaries.

B. Solving m-TSP for Efficient ASV Routing

Given all the closed curve contours generated from the previous step, Algorithm 1 provides a methodology to output enhanced paths for all the robots T, such that both $\sum_{i=1}^{m} |\tau_i|$ and $\mathrm{Var}(|\tau_i|)$ are minimized.

D, the distance matrix constructed using the free shortest path distances of the sequences of points representing the

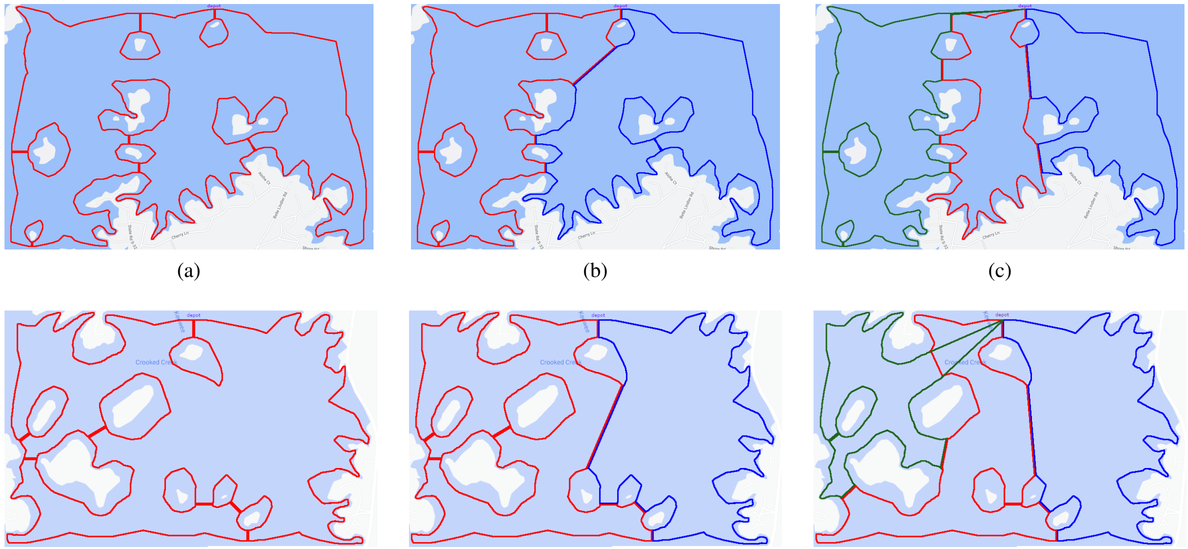

(a) (b) (c)
(d) (e) (f)

Fig. 3: Visualization of task assignments in Lake Murray and Lake Keowee. Different colors represent individual ASVs. Lake Murray: (a) Single ASV, (b) Dual ASVs, (c) Tri-ASVs. Lake Keowee: (d) Single ASV, (e) Dual ASVs, (f) Tri-ASVs. Note the location of the depot on each figure.

curves C_i, determined using the wavefront transform. m denotes the number of vehicles; with $m = 1$, the problem reduces to TSP.

After evaluating various combinations of heuristics and metaheuristics, it was discerned that our problem formulation yielded the most efficient results when employing work by Christofides et al. [37] for the initial solution generation and the Guided Local Search (GLS) by Voudouris et al. [38] as the metaheuristic.

A vital aspect to consider is the preservation of individual trajectories generated for circumnavigating all the islands. The parent trajectory, with landmass outside it excluding the islands, differs from the trajectories around islands. Unlike the parent trajectory, where the ASV can traverse freely, the ASV cannot navigate across the interior of the island paths without possibly colliding with the circumnavigated island.

	1 ASV	**2 ASVs**	**3 ASVs**
min(τ)	25.9 km	13.20 km	8.10 km
max(τ)	25.9 km	13.50 km	10.70 km
avg(τ)	25.9 km	13.35 km	9.20 km
total(τ)	25.9 km	26.70 km	27.6 km

TABLE I: Quantitative data of Fig. 3 (a), (b) (c)

	1 ASV	**2 ASVs**	**3 ASVs**
min(τ)	22.4 km	10.2 km	9.2 km
max(τ)	22.4 km	13.6 km	10.0 km
avg(τ)	22.4 km	11.9 km	9.5 km
total(τ)	22.4 km	23.8 km	28.5 km

TABLE II: Quantitative data of Fig. 3 (d), (e) (f)

The system was extensively tested on selected areas of different lakes with different configuration. Fig. 3 shows the generated paths for the selected areas of Lake Murray and Lake Keowee. Quantitative data in Table I and Table II show a direct correlation between the increase in number of robots deployed and the decrease in total deployment time.

V. EXPERIMENTAL RESULTS

A. Platform

An ASV developed at the University of South Carolina [3], based on a modified Mokai Es-Kape2 motor watercraft, the Jetyak, was used for the field experiments. The Jetyak comes with a 7HP OHC horizontal engine that reaches speeds up to 22.5 km/h and has a deployment time of over eight hours. The Jetyak has a Sonar side scanner for collecting bathymetry data, a YSI EXO2 multiparameter sonde[10], and the OTT Nitrate ecoN UV Nitrate sensor[11] to collect water quality samples near the surface along the path. The ES-Kape's factory pulse width modulated (PWM) controlled servo system allows seamless integration with a Pixhawk flight control system and onboard control through an Intel UP single-board computer running ROS.

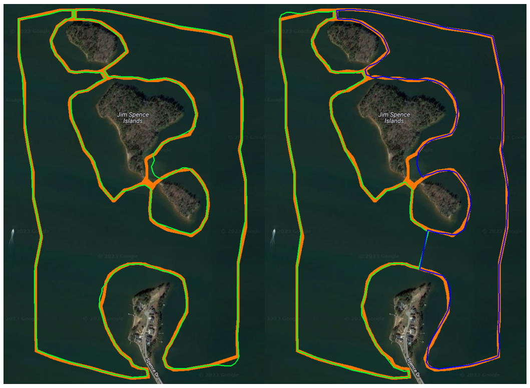
B. Experimental Procedure in the Field

The primary objective of experiments are to evaluate the ASV trajectories produced under natural conditions and to gather water quality data along these trajectories. Experiments were conducted on Lake Murray, SC, USA (see Fig. 4) in two distinct setups: (a) employing a single ASV and (b) deploying dual ASVs. Both setups shared a common starting point. The dual ASV deployment reduced the maximum distance (τ) value from 12.1 km with one ASV to 6.5 km cutting the deployment time by half, as shown in Table III. The increase in total τ from 12.1 km to 12.6 km is a mere 4% increase in total traversal distance.

Water quality measurements revealed a mean temperature of $27.8°C$. pH values were documented within a range of

	1 ASV	2 ASVs
min(τ)	12.1 km	6.1 km
max(τ)	12.1 km	6.5 km
avg(τ)	12.1 km	6.3 km
total(τ)	12.1 km	12.6 km

TABLE III: Quantitative data of Lake Murray Field Deployment



(a) (b)

Fig. 4: Lake Murray Field Experiment trajectories. (a) single ASV. (b) Dual ASV.

8.11 to 8.45. Dissolved Oxygen saturation remained close to 100%, with negligible variations. Chlorophyll RFU measurements ranged from 0.68 to 0.99. Nitrate levels were observed between 0.473 and 5.0 mg N L^{-1}. A detailed graphical representation of these observations is presented in Figure 5.

VI. Conclusion

This paper presents a complete system solution to plan efficient trajectories for multiple ASVs. The produced paths guide ASVs through large water expanses, including areas with islands, for comprehensive water quality sampling to observe transient marine phenomena. Our strategy hinges on the skeleton-of-skeleton algorithm and extends it to multi-agent systems. We have conducted extensive field deployments, traversing large regions of a lake and collecting water quality measurements with the YSI EXO2 Multiparameter sonde and the OTT Nitrate sensors.

However, the system currently relies heavily on offline maps for trajectory planning and sampling location accuracy in the field. To address this limitation, future work will focus on integrating online strategies to manage uncertainties in location and dynamic obstacles.

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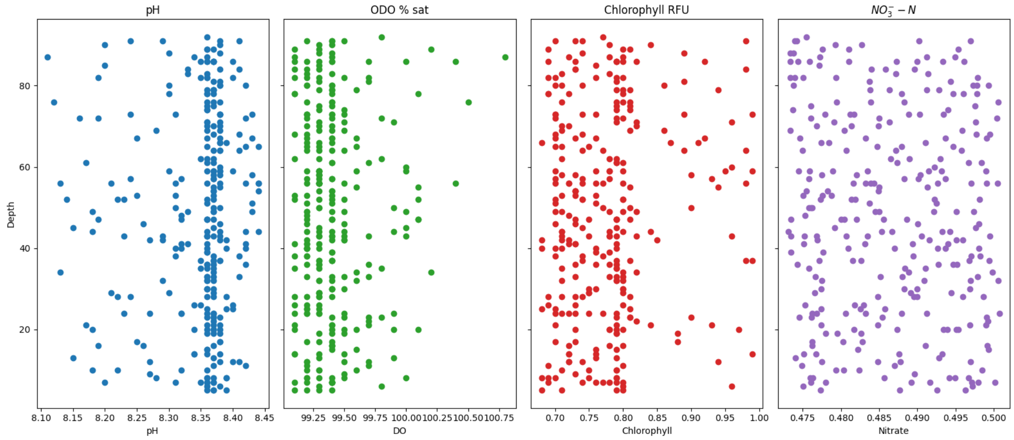

Fig. 5: Water quality measures from the field deployment at Lake Murray, SC, USA.

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